Solving Systems of Equations - Word Problems

1. The sum of two numbers is 36 and their difference is 24. Find the numbers.

1st Number $= x = 30$
2nd Number $= y = 6$

$x + y = 36$
$x - y = 24$
_____
$\dfrac{2x}{2} = \dfrac{60}{2}$

$x = 30$

$x + y = 36$
$30 + y = 36$
$-30 \qquad -30$
$\qquad y = 6$

2. The sum of two numbers is 78. The larger number is three more than two times the smaller number.
   Find the numbers. $x \quad = \quad 3 + 2y$

Larger Number $= x = 53$
Smaller Number $= y = 25$

$x + y = 78$
$x = \boxed{3 + 2y}$

$3 + 2y + y = 78$
$3 + 3y = 78$
$-3 \qquad -3$
$\dfrac{3y}{3} = \dfrac{75}{3}$
$y = 25$

$x + y = 78$
$x + 25 = 78$
$\quad -25 \ -25$
$x = 53$

3. The difference between two numbers is 34. The larger exceeds three times the smaller by four. Find the numbers.

$x = +4 + 3y$

Larger Number $= x = 49$
Smaller Number $= y = 15$

$x - y = 34$
$x = 4 + 3y$

$4 + 3y - y = 34$
$4 + 2y = 34$
$-4 \qquad -4$
$\dfrac{2y}{2} = \dfrac{30}{2}$
$y = 15$

$x - y = 34$
$x - 15 = 34$
$\quad +15 \quad +15$
$x = 49$

4. Laura bought six belts and eight jackets for $140. A week later, at the same prices, she bought nine belts and six jackets for $132. Find the price of the belt and jacket.

Belt $= x = \$6$
Jacket $= y = \$13$

$3(6x + 8y = 140) \rightarrow 18x + 24y = 420$
$-2(9x + 6y = 132) \rightarrow -18x - 12y = -264$

$\dfrac{12y}{12} = \dfrac{156}{12}$

$y = 13$

$6x + 8y = 140$
$6x + 8(13) = 140$
$6x + 104 = 140$
$\quad -104 \quad -104$
$\dfrac{6x}{6} = \dfrac{36}{6}$
$x = 6$

5. Tickets for a high school dance cost $1.00 each if purchased in advance but $1.50 each if purchased at the door. If 100 tickets were sold and $120 was collected, how many tickets were sold in advance and how many were sold at the door?

Advance = x = 60 tickets
Door = y = 40 tickets

$x + y = 100$
$\$1x + \$1.50y = \$120$ → $1x + 1.5y = 120$
$10x + 15y = 1200$

$-10(x + y = 100)$ → $-10x - 10y = -1000$
$10x + 15y = 1200$ → $10x + 15y = 1200$

$$\frac{5y}{5} = \frac{200}{5} \quad y = 40$$

$x + y = 100$
$x + 40 = 100$
$-40 \quad -40$
$x = 60$

6. The perimeter of a rectangle is 50 cm. The length is 9 cm more than the width. Find the length and width of the rectangle.

$x = 9 + y$

Length = x = 17 cm
Width = y = 8 cm

y [P = 50] y

$P = 2x + 2y$
* $2x + 2y = 50$
* $x = (9 + y)$

$2(9+y) + 2y = 50$
$18 + 2y + 2y = 50$
$18 + 4y = 50$
$-18 \qquad -18$
$$\frac{4y}{4} = \frac{32}{4}$$
$y = 8$

$x = 9 + y$
$x = 9 + 8$
$x = 17$

7. Bri received $1,400 for her birthday. She invested some at 5% and the rest at 8%. Her total annual income from both investments was $100. Find the amount she invested at each rate.

5% Rate = x = $400
8% Rate = y = $1000

$x + y = 1400 \rightarrow y = \boxed{1400 - x}$

$5\% x + 8\% y = 100 \rightarrow .05x + .08y = 100$

$5x + 8y = 10,000$
$5x + 8(1400 - x) = 10,000$
$5x + 11200 - 8x = 10,000$
$-3x + 11,200 = 10,000$
$ -11,200 \quad -11,200$
$\dfrac{-3x}{-3} = \dfrac{-1200}{-3}$
$x = 400$

$y = 1400 - x$
$y = 1400 - 400$
$y = 1000$

8. 193 students are attending a competition 100 miles away. There are eight drivers available and two types of vehicles, buses and minivans. The buses seat 51 people each and the minivans seat eight people each. How many buses and minivans will be needed?

Buses = x = 3 buses
Minivans = y = 5 minivans

$x + y = 8 \rightarrow y = \boxed{8 - x}$
$51x + 8y = 193$

$51x + 8(8 - x) = 193$
$51x + 64 - 8x = 193$
$43x + 64 = 193$
$ -64 \quad -64$
$\dfrac{43x}{43} = \dfrac{129}{43}$
$x = 3$

$y = 8 - x$
$y = 8 - 3$
$y = 5$

9. Kelly combines a 25% copper alloy and a 50% copper alloy to create 40 kg of a 45% copper alloy. How many kg of each does she need?

25% Alloy $= x = 8\ kg$
50% Alloy $= y = 32\ kg$

$x + y = 40 \longrightarrow y = 40 - x$
$25\% x + 50\% y = 45\%(40) \to .25x + .50y = 18$

$25x + 50y = 1800$
$25x + 50(40 - x) = 1800$
$25x + 2000 - 50x = 1800$
$-25x + 2000 = 1800$
$\quad\quad\quad -2000\quad -2000$
$\dfrac{-25x}{-25} = \dfrac{-200}{-25}$
$\quad\quad x = 8$

$y = 40 - x$
$y = 40 - 8$
$y = 32$

10. Danielle is four years older than her brother Andrew. Six years ago, she was twice as old as he was. How old are they now?

Danielle $= x = 14$ years old
Andrew $= y = 10$ years old

	6 years ago	Now
D	$x - 6$	x
A	$y - 6$	y

$x = 4 + y$
$x - 6 = 2(y - 6)$

$4 + y - 6 = 2(y - 6)$
$y - 2 = 2y - 12$
$-y \quad\quad -y$
$-2 = y - 12$
$+12 \quad\quad +12$
$10 = y$

$x = 4 + y$
$x = 4 + 10$
$x = 14$

$$D = RT$$

11. A canoe traveling with the current takes 2 hours to travel a distance of 24 miles. On the return trip, it takes 3 hours traveling against the current. Find the speed and current of the canoe in still water.

Speed = $x = 10$ mph
Current = $y = 2$ mph

To
D = 24 m
T = 2 h
R = x + y

From
D = 24 m
T = 3 h
R = x - y

$24 = (x+y)2$
$\dfrac{24}{2} = \dfrac{2x}{2} + \dfrac{2y}{2}$
$12 = x + y$

$24 = (x-y)3$
$\dfrac{24}{3} = \dfrac{3x}{3} - \dfrac{3y}{3}$
$8 = x - y$

$x + y = 12$
$x - y = 8$
$\dfrac{2x}{2} = \dfrac{20}{2}$
$x = 10$

$12 = x + y$
$12 = 10 + y$
$-10 \quad -10$
$y = 2$

12. Michael has 48 coins consisting of quarters and nickels and the total value is $6.00. How many quarters and nickels does he have?

Quarters = $x = 18$ Quarters
Nickels = $y = 30$ Nickels

$x + y = 48$ → $y = \boxed{48 - x}$
$.35x + .05y = 6.00$ $25x + 5y = 600$

$25x + 5(48 - x) = 600$
$25x + 240 - 5x = 600$
$20x + 240 = 600$
$\quad\quad -240 \quad -240$

$\dfrac{20x}{20} = \dfrac{360}{20}$
$x = 18$

$y = 48 - x$
$y = 48 - 18$
$y = 30$