# Solving Rational Equations

Step 1: Find the LCD and multiply each fraction by what is missing in the LCD.
Step 2: Cancel denominators.
Step 3: Solve for the variable.
Step 4: Check your answers.

1. Solve each rational equation.

a) $\dfrac{12}{12}\cdot\dfrac{x-1}{x}\cdot\dfrac{4}{4}+\dfrac{7}{3x}\cdot\dfrac{3}{3}=\dfrac{9}{4x}\cdot\dfrac{3}{3}$

$LCD = 12x$

$\dfrac{12(x-1)}{12x}+\dfrac{28}{12x}=\dfrac{27}{12x}$

$12(x-1)+28=27$

$12x-12+28=27$

$12x+16=27$

$\quad -16 \quad -16$

$\dfrac{12x}{12}=\dfrac{11}{12}$

$\boxed{x=\dfrac{11}{12}}$

**Check**

$x=\dfrac{11}{12}$

$X: \dfrac{11}{12}$

$3x: \ 3\left(\dfrac{11}{12}\right)=\dfrac{33}{12}$

$4x: \ 4\left(\dfrac{11}{12}\right)=\dfrac{44}{12}$

b) $\dfrac{6}{t-1}+2=\dfrac{12}{t^2-1}$

$\dfrac{(t+1)\cdot 6}{(t+1)\cdot t-1}+\dfrac{2}{1\cdot(t+1)(t-1)}\dfrac{(t-1)(t+1)}{}=\dfrac{12}{(t+1)(t-1)}$

$LCD = (t-1)(t+1)$

$\dfrac{6(t+1)}{(t+1)(t-1)}+\dfrac{2(t-1)(t+1)}{(t+1)(t-1)}=\dfrac{12}{(t+1)(t-1)}$

FOIL

$6(t+1)+2(t-1)(t+1)=12$

$6t+6+2(t^2+t-t-1)=12$

$6t+6+2(t^2-1)=12$

$6t+6+2t^2-2=12$

$2t^2+6t+4=12$

$\quad -12-12$

$\dfrac{2t^2}{2}+\dfrac{6t}{2}-\dfrac{8}{2}=\dfrac{0}{2}$

$t^2+3t-4=0$

$(t+4)(t-1)=0$

**check**

$t-1: \ 1-1=0$

$t^2-1: \ 1^2-1=0$

$$(t+4)(t-1)=0$$

$$t+4=0 \qquad t-1=0$$

$$\boxed{t=-4} \qquad \cancel{t=1}$$

c) $\dfrac{x}{x-4}=6+\dfrac{x}{x-4}$

$$\dfrac{x}{\cancel{x-4}} = \dfrac{6(x-4)}{\cancel{x-4}} + \dfrac{x}{\cancel{x-4}}$$

$$LCD = (x-4)$$

$$x = 6(x-4)+x$$
$$x = 6x-24+x$$
$$x = 7x-24$$
$$-7x \qquad -7x$$

Check:

$$x-4: \ 4-4=0$$

$$\dfrac{-6x}{-6} = \dfrac{-24}{-6}$$

$$\cancel{x=4} \qquad \boxed{\text{No solution}}$$

d) $\dfrac{10}{m+3}-\dfrac{3}{5}=\dfrac{10m+1}{3m+9}$

$$\dfrac{5 \cdot 10}{15 \cdot m+3} - \dfrac{3 \cdot 3(m+3)}{5 \cdot 3(m+3)} = \dfrac{10m+1 \cdot 5}{3(m+3) \cdot 5}$$

$$LCD=15(m+3)$$

$$\dfrac{150}{\cancel{15(m+3)}} - \dfrac{9(m+3)}{\cancel{15(m+3)}} = \dfrac{5(10m+1)}{\cancel{15(m+3)}}$$

$$150 - 9(m+3) = 5(10m+1)$$
$$150-9m-27 = 50m+5$$
$$-9m+123 = 50m+5$$
$$-50m \ -123 \quad -50m \ -123$$
$$\dfrac{-59m}{-59} = \dfrac{-118}{-59}$$

$$\boxed{m=2}$$

Check

$$m+3: \ 2+3=5$$

e) $\dfrac{3}{y-2} - \dfrac{6}{y^2-2y} = 1$

$\dfrac{y \cdot 3}{y \cdot y-2} - \dfrac{6}{y(y-2)} = \dfrac{1 \cdot y(y-2)}{1 \cdot y(y-2)}$

$LCD = y(y-2)$

$\dfrac{3y}{y(y-2)} - \dfrac{6}{y(y-2)} = \dfrac{y(y-2)}{y(y-2)}$

$3y - 6 = y(y-2)$

$3y - 6 = y^2 - 2y$

$-3y + 6 \qquad -3y + 6$

$0 = y^2 - 5y + 6$

$0 = (y-2)(y-3)$

$y - 2 = 0 \qquad\qquad y - 3 = 0$

$+2 \; +2 \qquad\qquad +3 \; +3$

$\cancel{y=2} \qquad\qquad \boxed{y = 3}$

Check

$y(y-2): \quad 2(2-2)$

$2(0)$

$0$