

# Multiplying and Dividing Radical Expressions

## Multiplication

$$\sqrt{a} \cdot \sqrt{b} = \sqrt{ab}$$

1. Simplify each radical expression.

a)  $(3\sqrt{6})(8\sqrt{5})$

$$24\sqrt{30}$$

$$\begin{array}{c} 30 \\ \wedge \\ 2 \quad 15 \\ \quad \wedge \\ \quad 3 \quad 5 \\ 2 \cdot 3 \cdot 5 \end{array}$$

b)  $(-2\sqrt{12})(7\sqrt{8})$

$$\begin{aligned} & -14\sqrt{96} \\ & -14\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3} \\ & -14 \cdot 2 \cdot 2 \sqrt{6} \\ & -56\sqrt{6} \end{aligned}$$

$$\begin{array}{c} 96 \\ \wedge \\ 2 \quad 48 \\ \quad \wedge \\ \quad 2 \quad 24 \\ \quad \quad \wedge \\ \quad \quad 2 \quad 12 \\ \quad \quad \quad \wedge \\ \quad \quad \quad 2 \quad 6 \\ \quad \quad \quad \quad \wedge \\ \quad \quad \quad \quad 2 \quad 3 \\ 2 \cdot 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \end{array}$$

c)  $\sqrt{48} \cdot \sqrt{3} = \sqrt{48 \cdot 3}$

$$\sqrt{2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \cdot 3} = 2 \cdot 2 \cdot 3 = 12$$

$$\begin{array}{c} 48 \\ \wedge \\ 2 \quad 24 \\ \quad \wedge \\ \quad 2 \quad 12 \\ \quad \quad \wedge \\ \quad \quad 2 \quad 6 \\ \quad \quad \quad \wedge \\ \quad \quad \quad 2 \quad 3 \\ 2 \cdot 2 \cdot 2 \cdot 2 \cdot 3 \end{array}$$

d)  $(5\sqrt{3})^2 = 5\sqrt{3} \cdot 5\sqrt{3}$

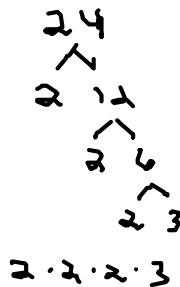
$$= 25\sqrt{9} = 25 \cdot 3 = 75$$

e)  $\sqrt{2}(8+\sqrt{12})$

$$8\sqrt{2} + \sqrt{24}$$

$$8\sqrt{2} + \sqrt{2 \cdot 2 \cdot 2 \cdot 3}$$

$$8\sqrt{2} + 2\sqrt{6}$$



f)  $(3-\sqrt{6})(4+\sqrt{8})$

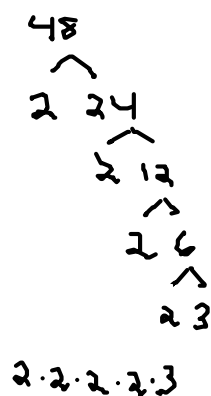
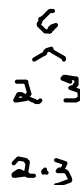
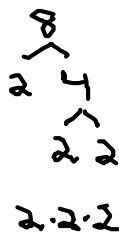
FOIL

$$12 + 3\sqrt{8} - 4\sqrt{6} - \sqrt{48}$$

$$12 + 3\sqrt{2 \cdot 2 \cdot 2} - 4\sqrt{2 \cdot 3} - \sqrt{2 \cdot 2 \cdot 2 \cdot 3}$$

$$12 + 3 \cdot 2\sqrt{2} - 4\sqrt{6} - 2 \cdot 2\sqrt{3}$$

$$12 + 6\sqrt{2} - 4\sqrt{6} - 4\sqrt{3}$$



g)  $(5+\sqrt{7})(5-\sqrt{7})$

FOIL

$$25 - 5\sqrt{7} + 5\sqrt{7} - \sqrt{49}$$

$$25 - 7 = 18$$

Division

$$\frac{\sqrt{a}}{\sqrt{b}} = \sqrt{\frac{a}{b}}$$

2. Simplify each radical expression.

$$\text{a) } \sqrt{\frac{36}{49}} = \frac{\sqrt{36}}{\sqrt{49}} = \boxed{\frac{6}{7}}$$

$$\text{b) } \sqrt{\frac{7}{16}} = \frac{\sqrt{7}}{\sqrt{16}} = \boxed{\frac{\sqrt{7}}{4}}$$

3. Rationalize each denominator.

$$\text{a) } \sqrt{\frac{1}{5}} = \frac{\sqrt{1}}{\sqrt{5}} = \frac{1}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}}{\sqrt{25}} = \boxed{\frac{\sqrt{5}}{5}}$$

$$b) \frac{3+\sqrt{6}}{\sqrt{10}} \cdot \frac{\sqrt{10}}{\sqrt{10}} = \frac{\sqrt{10}(3+\sqrt{6})}{\sqrt{100}} = \frac{3\sqrt{10} + \sqrt{60}}{10}$$

$$\frac{3\sqrt{10} + \sqrt{2 \cdot 3 \cdot 5}}{10} = \frac{3\sqrt{10} + 2\sqrt{15}}{10}$$

$$\begin{array}{c} 10 \\ \wedge \\ 2 \quad 5 \\ 2 \cdot 5 \end{array}$$

$$\begin{array}{c} 60 \\ \wedge \\ 2 \quad 30 \\ \quad \wedge \\ \quad 2 \quad 15 \\ \quad \quad \wedge \\ \quad \quad 3 \quad 5 \\ 2 \cdot 2 \cdot 3 \cdot 5 \end{array}$$

$$c) \frac{6-\sqrt{18}}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}(6-\sqrt{18})}{\sqrt{9}} = \frac{6\sqrt{3} - \sqrt{54}}{3}$$

$$\frac{6\sqrt{3} - \sqrt{2 \cdot 3 \cdot 3 \cdot 3}}{3} = \frac{6\sqrt{3} - 3\sqrt{6}}{3}$$

$$\begin{array}{c} 54 \\ \wedge \\ 2 \quad 27 \\ \quad \wedge \\ \quad 3 \quad 9 \\ \quad \quad \wedge \\ \quad \quad 3 \quad 3 \\ 2 \cdot 3 \cdot 3 \cdot 3 \end{array}$$

$$\frac{2\sqrt{3} - \sqrt{6}}{1}$$

$$2 \cdot 3 \cdot 3 \cdot 3$$

$$d) \frac{\sqrt{15}-\sqrt{10}}{\sqrt{5}} \cdot \frac{\sqrt{5}}{\sqrt{5}} = \frac{\sqrt{5}(\sqrt{15}-\sqrt{10})}{\sqrt{25}} = \frac{\sqrt{75}-\sqrt{50}}{5}$$

$$\frac{\sqrt{3 \cdot 5 \cdot 5} - \sqrt{2 \cdot 5 \cdot 5}}{5} = \frac{\sqrt{3} \cdot \sqrt{5} - \sqrt{2} \cdot \sqrt{5}}{5}$$

$\begin{matrix} 75 & & 50 \\ \wedge & & \wedge \\ 3 & 25 & 2 & 25 \\ & \wedge & & \wedge \\ & 5 & 5 & & 5 & 5 \\ & & 3 \cdot 5 \cdot 5 & & 2 \cdot 5 \cdot 5 \end{matrix}$

$$\boxed{\sqrt{3}-\sqrt{2}}$$

$$e) \frac{3}{3+\sqrt{6}} \cdot \frac{3-\sqrt{6}}{3-\sqrt{6}} = \frac{3(3-\sqrt{6})}{(3+\sqrt{6})(3-\sqrt{6})} = \frac{9-3\sqrt{6}}{9-3\sqrt{6}+3\sqrt{6}-\sqrt{36}}$$

$$= \frac{9-3\sqrt{6}}{9-6} = \frac{3-3\sqrt{6}}{3} = \boxed{3-\sqrt{6}}$$

$$f) \frac{4+\sqrt{2}}{1+\sqrt{3}} \cdot \frac{1-\sqrt{3}}{1-\sqrt{3}} = \frac{(4+\sqrt{2})(1-\sqrt{3})}{(1+\sqrt{3})(1-\sqrt{3})} = \frac{4-4\sqrt{3}+\sqrt{2}-\sqrt{6}}{1-\sqrt{3}+\sqrt{3}-\sqrt{9}}$$

$$\frac{4-4\sqrt{3}+\sqrt{2}-\sqrt{6}}{1-3} = \boxed{\frac{4-4\sqrt{3}+\sqrt{2}-\sqrt{6}}{-2}}$$

$$g) \frac{\sqrt{5} + \sqrt{6}}{\sqrt{3} - \sqrt{2}} \cdot \frac{\sqrt{3} + \sqrt{2}}{\sqrt{3} + \sqrt{2}} = \frac{(\sqrt{5} + \sqrt{6})(\sqrt{3} + \sqrt{2})}{(\sqrt{3} - \sqrt{2})(\sqrt{3} + \sqrt{2})}$$

$$\frac{\sqrt{15} + \sqrt{10} + \sqrt{18} + \sqrt{12}}{\sqrt{9} + \sqrt{6} - \sqrt{6} - \sqrt{4}} = \frac{\sqrt{15} + \sqrt{10} + \sqrt{2 \cdot 3 \cdot 3} + \sqrt{2 \cdot 2 \cdot 3}}{3 - 2}$$

$$\begin{array}{c} 18 \\ \wedge \\ 2 \quad 9 \\ \wedge \\ 3 \quad 3 \end{array}$$

$$2 \cdot 3 \cdot 3$$

$$\begin{array}{c} 12 \\ \wedge \\ 2 \quad 6 \\ \wedge \\ 2 \quad 3 \end{array}$$

$$2 \cdot 2 \cdot 3$$

$$= \frac{\sqrt{15} + \sqrt{10} + 3\sqrt{2} + 2\sqrt{3}}{1}$$