Solving Radical Equations

Step 1: Isolate the radical symbol and square both sides of the equation.
Step 2: Solve for the variable.
Step 3: Check your answer.

1. Solve each equation.

a) $\left(\sqrt{x+2}\right)^2 = \left(5\right)^2$

$x+2=25$
$-2 \quad -2$
$\boxed{x=23}$

Check $x=23$

$\sqrt{x+2}=5$
$\sqrt{23+2}=5$
$\sqrt{25}=5$
$5=5$ ✓

b) $\left(\sqrt{3x-5}\right)^2 = \left(4\right)^2$

$3x-5=16$
$+5 \quad +5$

$\frac{3x}{3}=\frac{21}{3}$

$\boxed{x=7}$

Check $x=7$

$\sqrt{3x-5}=4$
$\sqrt{3\cdot 7-5}=4$
$\sqrt{16}=4$
$4=4$ ✓

c) $\left(\sqrt{x+2}\right)^2=(-5)^2$

$x+2=25$
$\quad -2 \quad -2$
$x=23$

extraneous solution
No solution

Check $x=23$

$\sqrt{x+2}=-5$
$\sqrt{23+2}=-5$
$\sqrt{25}=-5$
$5=-5$ X

d) $\left(\sqrt{x+6}\right)^2=(x)^2$

$x+6=x^2$
$-x-6 \quad -x-6$

$0=x^2-x-6$

$1\cdot 6$
$\boxed{2\cdot 3}$

$0=(x+2)(x-3)$
$x+2=0 \qquad x-3=0$
$-2 \; -2 \qquad +3 \; +3$
$x=-2 \qquad \boxed{x=3}$

Check $x=-2$

$\sqrt{x+6}=x$
$\sqrt{-2+6}=-2$
$\sqrt{4}=-2$
$2=-2$ X extraneous solution

Check $x=3$

$\sqrt{x+6}=x$
$\sqrt{3+6}=3$
$\sqrt{9}=3$
$3=3$ ✓

e) $\left(\sqrt{42-x}\right)^2=(x)^2$

$42-x=x^2$
$-42+x \quad -42+x$

$0=x^2+x-42$

$1\cdot 42$
$2\cdot 21$
$3\cdot 14$
$\boxed{6\cdot 7}$

$0=(x-6)(x+7)$
$x-6=0 \qquad x+7=0$
$+6 \; +6 \qquad -7 \; -7$

Check $x=6$

$\sqrt{42-x}=x$
$\sqrt{42-6}=6$
$\sqrt{36}=6$
$6=6$ ✓

Check $x=-7$

$\sqrt{42-x}=x$
$\sqrt{42--7}=-7$
$\sqrt{49}=-7$
$7=-7$ X

$x=-7$ extraneous solution

$+6 \quad +6 \qquad -7 \quad -7$

$\boxed{x=6} \qquad x=-7$

f) $\left(\sqrt{x-1}\right)^2=(x-1)^2$

$x-1=(x-1)^2$

$x-1=(x-1)(x-1)$

$x-1=x^2-x-x+1$

$x-1=x^2-2x+1$

$-x+1 \qquad -x+1$

$0=x^2-3x+2$

$2\cdot 1$

$0=(x-2)(x-1)$

$x-2=0 \qquad x-1=0$

$+2 \quad +2 \qquad +1 \quad +1$

$\boxed{x=2} \qquad \boxed{x=1}$

Check $x=2$

$\sqrt{x-1}=x-1$

$\sqrt{2-1}=2-1$

$\sqrt{1}=1$

$1=1$ ✓

Check $x=1$

$\sqrt{x-1}=x-1$

$\sqrt{1-1}=1-1$

$\sqrt{0}=0$

$0=0$ ✓

g) $\left(\sqrt{x^2+6x+2}\right)^2=(x+1)^2$

$x^2+6x+2=(x+1)^2$

$x^2+6x+2=(x+1)(x+1)$

$x^2+6x+2=x^2+x+x+1$

$\cancel{x^2}+6x+2=\cancel{x^2}+2x+1$

$-\cancel{x^2} \qquad -\cancel{x^2}$

$6x+2=2x+1$

$-2x \qquad -2x$

$4x+2=1$

$-2 \quad -2$

$\frac{4x}{4}=\frac{-1}{4}$

$\boxed{x=-1/4}$

Check $x=-\frac{1}{4}$

$\sqrt{x^2+6x+2}=x+1$

$\sqrt{(-1/4)^2+6(-1/4)+2}=-\frac{1}{4}+1$

$\sqrt{\frac{1}{16}-\frac{3}{2}+\frac{2}{1}}=-\frac{1}{4}+\frac{1\cdot 4}{1\cdot 4}$

LCD=4

$-\frac{1}{4}+\frac{4}{4}=\frac{3}{4}$

$-\frac{1}{4}\cdot -\frac{1}{4}=\frac{1}{16}$

$6\cdot -\frac{1}{4}=-\frac{6\div 2}{4\div 2}=\frac{-3}{2}$

$\frac{1}{16}\ \frac{8\cdot 3}{8\cdot 2}+\frac{2\cdot 16}{1\cdot 16}=\frac{1}{16}-\frac{24}{16}+\frac{32}{16}$

LCD = 16

$\frac{1-24+32}{16}=\frac{9}{16}$

$\sqrt{\frac{9}{16}}=\frac{3}{4}$

$\frac{3}{4}=\frac{3}{4}$ ✓