Radical Expressions (Simplifying, Adding, Subtracting, Multiplying, Dividing and Rationalizing the Denominator)

index

$\sqrt[n]{x}$

Radicand

$\sqrt[2]{25} = 5$

Square Root		Perfect Squares	
1		1	
2		4	
3		9	
4		16	
5		25	
6		36	
7	$7 \times 7 = 49$	$\sqrt{49}$	$= 7$
8		64	
9		81	
10		100	
11		121	
12		144	
13		169	
14		196	
15		225	

Cubes Root		Perfect Cubes	
1		1	
2		8	
3	$3 \times 3 \times 3 =$	27	$\sqrt[3]{27} = 3$
4		64	
5		125	
6		216	
7		343	
8		512	
9		729	
10		1000	

Directions: Find each of the following square roots.

1. $\sqrt{144} = \sqrt{12 \cdot 12} = \boxed{12}$

2. $-\sqrt{144} = -1\sqrt{12 \cdot 12} = -1 \cdot 12 = \boxed{-12}$

3. $\sqrt[2]{-144} = \boxed{\text{No solution}}$

4. $\sqrt[3]{-27} = \sqrt[3]{-3 \cdot -3 \cdot -3} = \boxed{-3}$

5. $\sqrt[2]{36x^8} = \sqrt{6 \cdot 6 \cdot x^4 \cdot x^4} = \boxed{6x^4}$

6. $\sqrt[4]{81x^{24}y^{16}} = \sqrt[4]{3 \cdot 3 \cdot 3 \cdot 3 \cdot x^6 \cdot x^6 \cdot x^6 \cdot x^6 \cdot y^4 \cdot y^4 \cdot y^4 \cdot y^4} = \boxed{3x^6y^4}$

$1^4 = 1$

$2^4 = 16$

$3^4 = 81$

Directions: Simplify each radical expression.

7. $\sqrt{50} = \sqrt{25\cdot 2} = \sqrt{25}\cdot\sqrt{2}$

$= \boxed{5\sqrt{2}}$

50
1·50
2·25
5·10

8. $4\sqrt{75} = 4\sqrt{25\cdot 3} = 4\sqrt{25}\cdot\sqrt{3}$

$= 4\cdot 5\sqrt{3}$

$= \boxed{20\sqrt{3}}$

75
1·75
3·25
5·15

9. $\sqrt[2]{a^2b^3} = a^1b^1\sqrt{b^1} = \boxed{ab\sqrt{b}}$

Q R
1 R 1
2) 3
-2
1

10. $\sqrt[2]{48x^4y^3} = x^2y^1\sqrt{16\cdot 3y^1}$

$= \boxed{4x^2y\sqrt{3y}}$

48
1·48
2·24
3·16
4·12
6·8

11. $\sqrt[2]{12x^7y^6} = x^3y^3\sqrt{4\cdot 3x^1}$

$= \boxed{2x^3y^3\sqrt{3x}}$

12
1·12
2·6
3·4

Q R
3 R 1
2) 7
-6
1

12. $2ab\sqrt[3]{40a^5b^4} = 2ab\cdot a^1b^1\sqrt[3]{8\cdot 5\cdot a^2b^1}$

$= 2a^2b^2\sqrt[3]{8\cdot 5a^2b}$

$= 2\cdot 2a^2b^2\sqrt[3]{5a^2b}$

$= \boxed{4a^2b^2\sqrt[3]{5a^2b}}$

40
1·40
2·20
4·10
5·8

13. $\sqrt[3]{54a^6b^2c^4} = a^2c^1\sqrt[3]{27\cdot 2\cdot b^2c^1}$

$= \boxed{3a^2c\sqrt[3]{2b^2c}}$

54
1·54
2·27
3·18
6·9

14. $\sqrt[5]{64x^8y^4z^{11}} = x^1z^2\sqrt[5]{32\cdot 2x^3y^4z^1}$

$= \boxed{2xz^2\sqrt[5]{2x^3y^4z}}$

64
1·64
2·32
4·16
8·8

$1^5 = 1$
$2^5 = 32$
$3^5 = 243$

Directions: Add or subtract the radical expressions and simplify your answer.

15. $5\sqrt{3}-4\sqrt{3}+6\sqrt{3}$ $= \boxed{7\sqrt{3}}$

16. $3\sqrt{8}+6\sqrt{18}$ $= 3\sqrt{4\cdot 2} + 6\sqrt{9\cdot 2}$

8: 1·8, 2·4

18: 1·18, 2·9, 3·6

$= 3\cdot 2\sqrt{2} + 6\cdot 3\sqrt{2}$

$= 6\sqrt{2} + 18\sqrt{2}$

$= \boxed{24\sqrt{2}}$

17. $7\sqrt[2]{75xy^3} - 4y\sqrt{12xy}$ $= 7y^1 \sqrt[2]{25\cdot 3x^1y^1} - 4y \sqrt[2]{4\cdot 3x^1y^1}$

75: 1·75, 3·25, 5·15

12: 1·12, 3·4, 2·6

$= 7\cdot 5y\sqrt{3xy} - 4y\cdot 2\sqrt{3xy}$

$= 35y\sqrt{3xy} - 8y\sqrt{3xy}$

$= \boxed{27y\sqrt{3xy}}$

18. $10\sqrt[3]{8a^4b^2} + 11a\sqrt[3]{27ab^2}$ $= 10\cdot 2 \sqrt[3]{a^4b^2} + 11a\cdot 3\sqrt[3]{a^1b^2}$

$= 20a\sqrt[3]{ab^2} + 33a\sqrt[3]{ab^2}$

$= \boxed{53a\sqrt[3]{ab^2}}$

19. $y\sqrt[3]{24x^5y^{1}}+3x\sqrt[3]{81x^2y^4}$ $= y\cdot x\sqrt[3]{8\cdot 3x^2y} + 3xy\sqrt[3]{27\cdot 3x^2y}$

24	81
$1\cdot 24$	$1\cdot 81$
$2\cdot 12$	$3\cdot 27$
$3\cdot 8$	$9\cdot 9$
$4\cdot 6$	

$= 2xy\sqrt[3]{3x^2y} + 3xy\cdot 3\sqrt[3]{3x^2y}$

$= 2xy\sqrt[3]{3x^2y} + 9xy\sqrt[3]{3x^2y}$

$= \boxed{11xy\sqrt[3]{3x^2y}}$

Directions: Multiply or divide the radical expressions and simplify your answer.

20. $\left(3\sqrt{5}\right)\left(2\sqrt{7}\right) = \boxed{6\sqrt{35}}$

35
$1\cdot 35$
$5\cdot 7$

21. $\sqrt{3}\left(2\sqrt{6}-5\sqrt{12}\right) = 2\sqrt{18} - 5\sqrt{36}$

18
$1\cdot 18$
$2\cdot 9$
$3\cdot 6$

$= 2\sqrt{9\cdot 2} - 5\cdot 6$

$= 2\cdot 3\sqrt{2} - 30$

$= \boxed{6\sqrt{2} - 30}$

F O

22. $\left(\sqrt{3}+\sqrt{5}\right)\left(4\sqrt{3}-\sqrt{5}\right) = 4\sqrt{9} - \sqrt{15} + 4\sqrt{15} - \sqrt{25}$

I

L

$= 4\cdot 3$

15

$1\cdot 15$

$3\cdot 5$

$= \underline{\underline{12}} - 1\sqrt{15} + 4\sqrt{15} - \underline{\underline{5}}$

$= \boxed{7 + 3\sqrt{15}}$

23. $\left(\sqrt{x}+2\right)^2 = (\sqrt{x} + 2)(\sqrt{x} + 2)$

$= \sqrt[2]{x^2} + 2\sqrt{\underline{x}} + 2\sqrt{\underline{x}} + 4$

$= \boxed{x + 4\sqrt{x} + 4}$

24. $\left(3\sqrt{x}-2\sqrt{y}\right)^2 = (3\sqrt{x} - 2\sqrt{y})(3\sqrt{x} - 2\sqrt{y})$

$= 9\sqrt{x^2} - 6\sqrt{\underline{xy}} - 6\sqrt{\underline{xy}} + 4\sqrt{y^2}$

$= \boxed{9x - 12\sqrt{xy} + 4y}$

F

25. $\left(\sqrt{6}+\sqrt{3}\right)\left(\sqrt{6}-\sqrt{3}\right) = \sqrt{36} - \sqrt{9}$

L

$= 6 - 3$

$= \boxed{3}$

26. $\sqrt{\frac{5}{4}} = \frac{\sqrt{5}}{\sqrt{4}} = \boxed{\frac{\sqrt{5}}{2}}$

27. $\sqrt{\frac{5}{6}} = \frac{\sqrt{5}}{\sqrt{6}} \cdot \frac{\sqrt{6}}{\sqrt{6}} = \frac{\sqrt{30}}{\sqrt{36}} = \boxed{\frac{\sqrt{30}}{6}}$

Rationalizing the denominator

30
1·30
2·15
3·10
5·6

28. $\frac{3}{\sqrt{2}+1} \cdot \frac{\sqrt{2}-1}{\sqrt{2}-1} = \frac{3(\sqrt{2}-1)}{(\sqrt{2}+1)(\sqrt{2}-1)} = \frac{3\sqrt{2}-3}{\sqrt{4}-1} = \frac{3\sqrt{2}-3}{2-1}$

F L

$= \frac{3\sqrt{2}-3}{1} = \boxed{3\sqrt{2}-3}$

29. $\frac{6}{\sqrt{5}-\sqrt{2}} \cdot \frac{\sqrt{5}+\sqrt{2}}{\sqrt{5}+\sqrt{2}} = \frac{6(\sqrt{5}+\sqrt{2})}{(\sqrt{5}-\sqrt{2})(\sqrt{5}+\sqrt{2})} = \frac{6\sqrt{5}+6\sqrt{2}}{\sqrt{25}-\sqrt{4}}$

F L

$= \frac{6\sqrt{5}+6\sqrt{2}}{5-2} = \frac{\overset{2}{\cancel{6}}\sqrt{5}+\overset{2}{\cancel{6}}\sqrt{2}}{\underset{1}{\cancel{3}}} = \boxed{2\sqrt{5}+2\sqrt{2}}$

30. $\dfrac{\sqrt{5}+3}{\sqrt{5}-2} \cdot \dfrac{\sqrt{5}+2}{\sqrt{5}+2} = \dfrac{(\sqrt{5}+3)(\sqrt{5}+2)}{(\sqrt{5}-2)(\sqrt{5}+2)}$

$= \dfrac{\sqrt{25}+2\sqrt{5}+3\sqrt{5}+6}{\sqrt{25}-4} = \dfrac{5+5\sqrt{5}+6}{5-4}$

$= \boxed{11+5\sqrt{5}}$