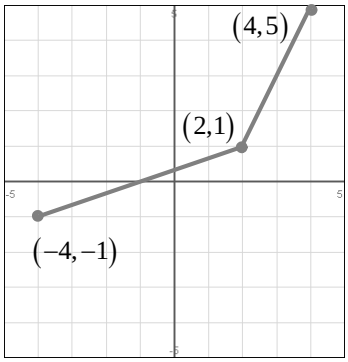
Shifting, Stretching and Reflecting Graphs

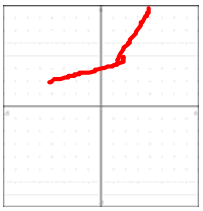
Transformations

$f(x)+c$	add c to each y value
$f(x)-c$	subtract c from each y value
$f(x+c)$	subtract c from each x value
$f(x-c)$	add c to each x value
$c \cdot f(x)$	multiply c by each y value
$f(c \cdot x)$	divide each x value by c

1. Use the graph to sketch each transformation.



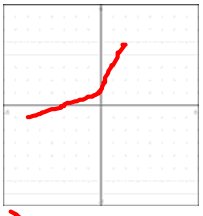
a) $f(x)+3$



add 3 to each y-value

$(-4,-1+3) = (-4,2)$
$(2,1+3) = (2,4)$
$(4,5+3) = (4,8)$

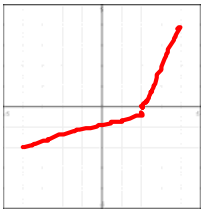
b) $f(x+2)$



subtract 2 from each x value

$(-4-2,-1) = (-6,-1)$
$(2-2,1) = (0,1)$
$(4-2,5) = (2,5)$

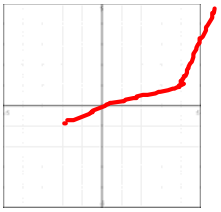
c) $f(x)-1$



subtract 1 from each y value

$(-4,-1-1) = (-4,-2)$
$(2,1-1) = (2,0)$
$(4,5-1) = (4,4)$

d) $f(x-2)$



add 2 to each x value

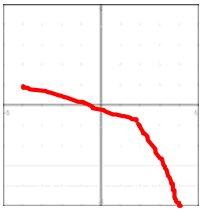
$(-4+2,-1)=(-2,-1)$
$(2+2,1)=(4,1)$
$(4+2,5)=(6,5)$

e) $2f(x)$

multiply each y value by 2

$(-4,-1\cdot 2)=(-4,-2)$
$(2,1\cdot 2)=(2,2)$
$(4,5\cdot 2)=(4,10)$

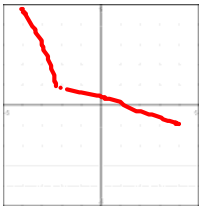
f) $-1f(x)$



multiply -1 by each y value

$(-4,-1\cdot -1)=(-4,1)$
$(2,1\cdot -1)=(2,-1)$
$(4,5\cdot -1)=(4,-5)$

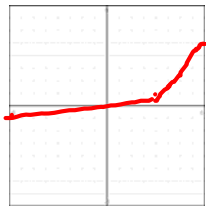
g) $f(-1x)$



divide each x value by -1

$(-4\div -1,-1)=(4,-1)$
$(2\div -1,1)=(-2,1)$
$(4\div -1,5)=(-4,5)$

h) $f\left(\frac{1}{2}x\right)$

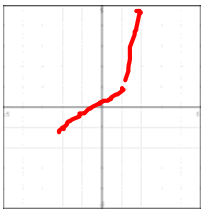


Divide each x value by ½

Multiply each x value by 2

$(-4\cdot 2,-1)=(-8,-1)$
$(2\cdot 2,1)=(4,1)$
$(4\cdot 2,5)=(8,5)$

i) $f(2x)$

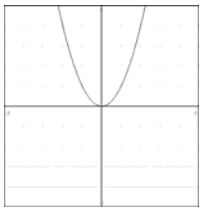


Divide each x value by 2

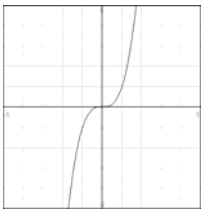
$(-4\div 2,-1)=(-2,-1)$
$(2\div 2,1)=(1,1)$
$(4\div 2,5)=(2,5)$

Parent Graphs

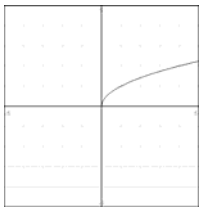
$y = x^2$



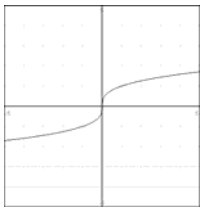
$y = x^3$



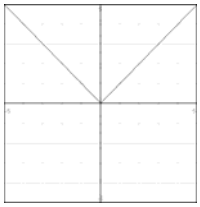
$y = \sqrt{x}$



$y = \sqrt[3]{x}$

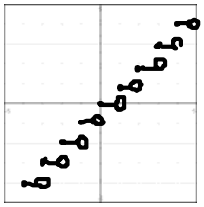


$y = |x|$



$y = [\![x]\!]$ – The Greatest Integer Function

– the greatest integer $\leq x$



$[\![2]\!] = 2$ $[\![\frac{1}{2}]\!] = 0$

$[\![-2]\!] = -2$ $[\![-\frac{1}{2}]\!] = -1$

Transformations

$f(x)+c$	shift graph up c units
$f(x)-c$	shift graph down c units
$f(x+c)$	shift graph left c units
$f(x-c)$	shift graph right c units
$-f(x)$	reflect graph over x-axis
$f(-x)$	reflect graph over y-axis
$f(c \cdot x)$	horizontal stretch if $0 < c < 1$, horizontal shrink if $c > 1$
$c \cdot f(x)$	vertical shrink if $0 < c < 1$, vertical stretch if $c > 1$

$y = x^3$

$y = x^3 + 2$

$y = x^3$

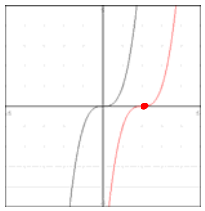
$y = x^3 - 2$

$y = x^3$

$y = (x+2)^3$

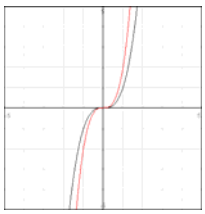
$y = x^3$

$y = (x-2)^3$



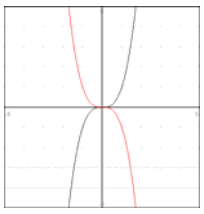
$y = x^3$

$y = 2x^3$



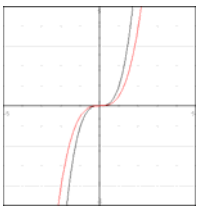
$y = x^3$

$y = -x^3$



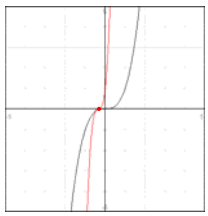
$y = x^3$

$y = \frac{1}{2}x^3$



$y = x^3$

$y = (3x+1)^3$



$3x + 1 = 0$

$-1 \quad -1$

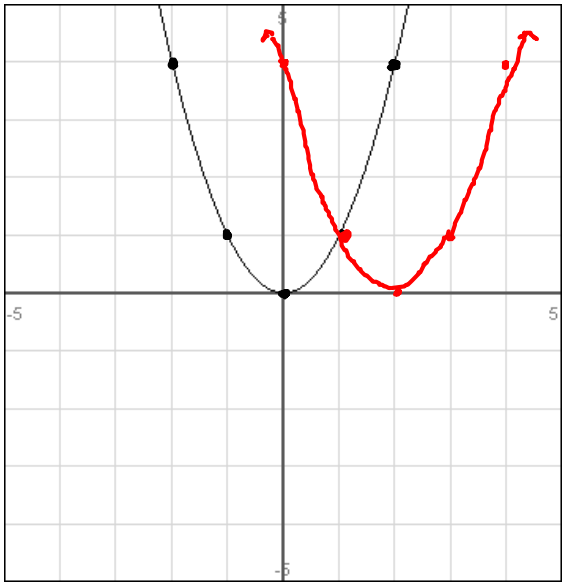
$\frac{3x}{3} = \frac{-1}{3}$

$x = -\frac{1}{3}$

2. Describe the transformation from the parent function and then sketch the graph.

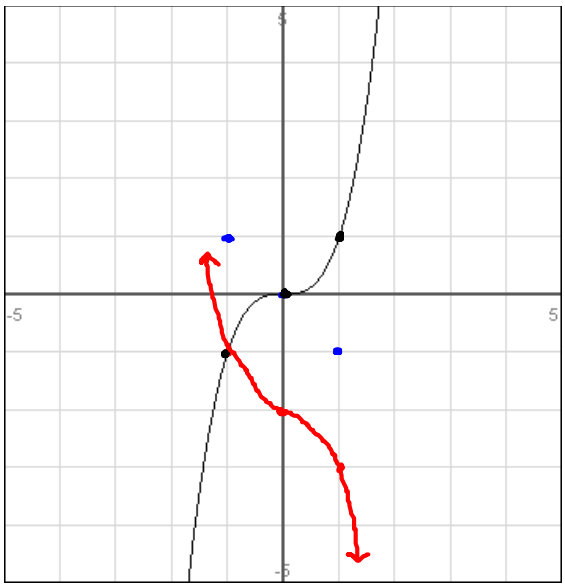
a) $f(x)=(x-2)^2$

shift graph 2 units to the right



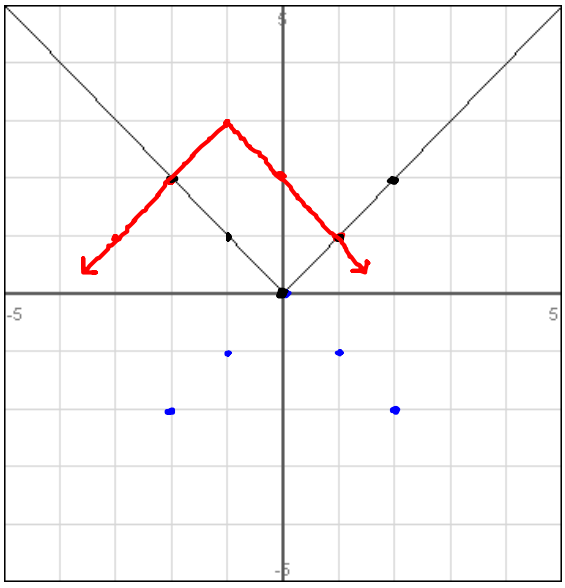
b) $f(x) = -x^3 - 2$

reflect graph over x-axis ←
shift down 2 units ←



c) $f(x) = 3 - |x+1|$

$f(x) = -|x+1| + 3$
reflect over x-axis ←
shift left 1 unit and ←
up 3 units



d) $f(x)=|-x+2|-1$

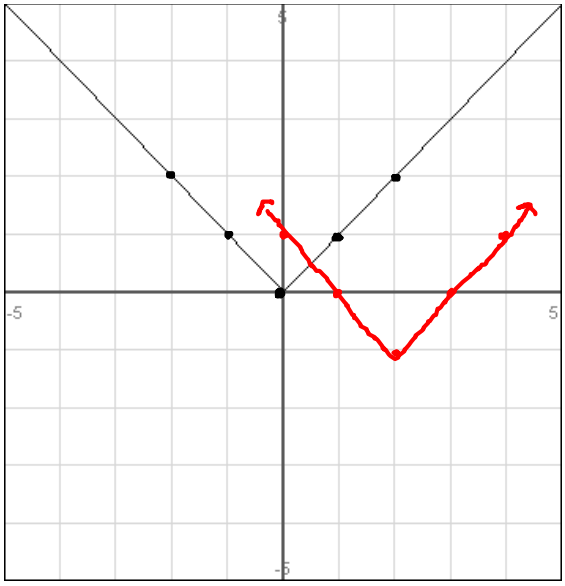
$-x+2=0$

$-2\ \ -2$

$\frac{-x}{-1}=\frac{-2}{-1}$

$x=2$

shift right 2 units & down 1 unit



e) $f(x)=2[\![x+3]\!]$

x	$2[\![x+3]\!]$	y
-5	$2[\![-2]\!]$	-4
-4	$2[\![-1]\!]$	-2
-3	$2[\![0]\!]$	0
-2	$2[\![1]\!]$	2
-1	$2[\![2]\!]$	4

shift to left 3 units
vertical stretch