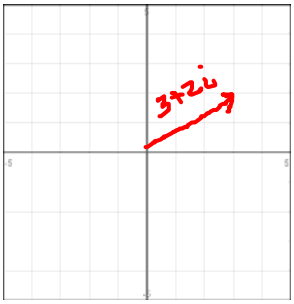
Complex Numbers

Complex Number - If a and b are real numbers, then the number $a+bi$ is a complex number.

$3+2i$

real, imaginary



$\sqrt{-1}=i$

$i^2=-1$

$i^3=-i$

$i^4=1$

1. Solve for a and b.

a) $a+bi=3-6i$

$a=3 \quad b=-6$

b) $(a-2)+3bi=5+9i$

$a-2=5 \quad 3b=9$

$+2 \quad +2 \quad \frac{3b}{3}=\frac{9}{3}$

$a=7 \quad b=3$

2. Write in standard form. $a+bi$

a) $6+\sqrt{-16}$

$6+4i$

b) 7

$7+0i$

c) $\sqrt{-20}=\sqrt{-4\cdot5}=2\sqrt{5}\,i$

$0+2\sqrt{5}\,i$

d) $-8i-4i^2 \qquad i^2=-1$

$-8i-4(-1)$

$-8i+4$

$4-8i$

3. Add or subtract the complex numbers and write the answer in standard form.

a) $(1-3i)+(-3+2i)$

$1-3i+-3+2i$

$\boxed{-2-i}$

b) $\left(-7-\sqrt{-72}\right)-\left(4-\sqrt{-98}\right)$

$\sqrt{-36}\ \sqrt{2}$ $\quad$ $\sqrt{-49}\ \sqrt{2}$

$6\sqrt{2}i$ $\quad$ $7\sqrt{2}i$

$(-7-6\sqrt{2}i)-(4-7\sqrt{2}i)$

$-7-6\sqrt{2}i-4+7\sqrt{2}i$

$\boxed{-11+\sqrt{2}i}$

4. Multiply the complex numbers and write the answer in standard form.

a) $\sqrt{-5}\cdot\sqrt{-10}$

$\sqrt{-1}\ \sqrt{5}$ $\quad$ $\sqrt{-1}\ \sqrt{10}$

$i\sqrt{5}\cdot i\sqrt{10}$

$i^2\sqrt{50}$ $\qquad i^2=-1$

$-1\sqrt{25\cdot 2}$

$\boxed{-5\sqrt{2}}$

b) $\left(\sqrt{-32}\right)^2=(\sqrt{-32})(\sqrt{-32})$

$\sqrt{-16}\ \sqrt{2}$

$4\sqrt{2}i$

$(4\sqrt{2}i)(4\sqrt{2}i)$

$16\sqrt{4}i^2$

$16(2)(-1)$

$\boxed{-32}$

c) $(6-2i)(-4+3i)$

$-24+18i+8i-6i^2$

$-24+26i-6(-1)$

$-24+26i+6$

$\boxed{-18+26i}$

d) $-6i(8-3i)$

$-48i+18i^2$

$-48i+18(-1)$

$-48i-18$

$\boxed{-18-48i}$

e) $\left(\sqrt{12}+\sqrt{10}i\right)\left(\sqrt{12}-\sqrt{10}i\right)$

F

$\sqrt{144}-\sqrt{100}\,i^2$

$12-10(-1)$

$12+10$

22

$22+0i$

f) $\left(5-\sqrt{-6}\right)\left(3+\sqrt{-12}\right)$

$\sqrt{-1}\ \sqrt{6}\qquad \sqrt{-4}\ \sqrt{3}$

$\sqrt{6}\,i\qquad 2\sqrt{3}\,i$

$(5-\sqrt{6}\,i)(3+2\sqrt{3}\,i)$

$15+10\sqrt{3}\,i-3\sqrt{6}\,i-2\sqrt{18}\,i^2$

$\sqrt{9}\ \sqrt{2}$

$-2(3)\sqrt{2}(-1)$

$6\sqrt{2}$

$15+10\sqrt{3}\,i-3\sqrt{6}\,i+6\sqrt{2}$

$(15+6\sqrt{2})+(10\sqrt{3}-3\sqrt{6})i$

g) $(3+6i)^2=(3+6i)(3+6i)$

$9+18i+18i+36i^2$

$9+36i+36(-1)$

$9+36i-36$

$-27+36i$

h) $(1-2i)^2-(1+2i)^2$

$(1-2i)(1-2i)-(1+2i)(1+2i)$

$(1-2i-2i+4i^2)-(1+2i+2i+4i^2)$

$(1-4i+4(-1))-(1+4i+4(-1))$

$(1-4i-4)-(1+4i-4)$

$(-3-4i)-(-3+4i)$

$-3-4i\ \ +3-4i$

$-8i$

$0-8i$

5. Divide and write the answer in standard form.

a) $-\frac{2}{i} \cdot \frac{i}{i} = \frac{-2i}{i^2}$

$\frac{-2i}{-1} = 2i$

$= \boxed{0+2i}$

b) $\frac{8-7i}{2i} \cdot \frac{i}{i} = \frac{i(8-7i)}{2i^2}$

$= \frac{8i-7i^2}{2i^2} = \frac{8i-7(-1)}{2(-1)}$

$= \frac{8i+7}{-2} = \frac{7+8i}{-2}$

$= \boxed{\frac{7}{-2} - 4i}$

c) $\frac{3}{2-i} \cdot \frac{2+i}{2+i} = \frac{3(2+i)}{(2-i)(2+i)}$

F L

$\frac{6+3i}{4-i^2} = \frac{6+3i}{4-(-1)} = \frac{6+3i}{5}$

$\boxed{\frac{6}{5} + \frac{3}{5}i}$

d) $\frac{3-2i}{5+6i} \cdot \frac{5-6i}{5-6i}$

$\frac{(3-2i)(5-6i)}{(5+6i)(5-6i)}$

F L

$\frac{15-18i-10i+12i^2}{25-36i^2}$

$\frac{15-28i+12(-1)}{25-36(-1)} = \frac{15-28i-12}{25+36}$

$\frac{3-28i}{61} = \boxed{\frac{3}{61} - \frac{28}{61}i}$

e) $\frac{3i(1-2i)}{4-i} = \frac{3i-6i^2}{4-i} = \frac{3i-6(-1)}{4-i} = \frac{3i+6}{4-i} \cdot \frac{4+i}{4+i} = \frac{(3i+6)(4+i)}{(4-i)(4+i)}$ (F, L)

$= \frac{12i+3i^2+24+6i}{16-i^2} = \frac{18i+3(-1)+24}{16-(-1)} = \frac{18i-3+24}{16+1}$

$= \frac{18i+21}{17} = \frac{21+18i}{17} = \boxed{\frac{21}{17} + \frac{18}{17}i}$

f) $\frac{2i}{3+i} - \frac{6}{3-i}$

$\frac{2i}{3+i} \cdot \frac{3-i}{3-i} = \frac{2i(3-i)}{(3+i)(3-i)} = \frac{6i-2i^2}{9-i^2} = \frac{6i-2(-1)}{9-(-1)} = \frac{6i+2}{10} = \frac{3i+1}{5}$ (F, L)

$\frac{6}{3-i} \cdot \frac{3+i}{3+i} = \frac{6(3+i)}{(3-i)(3+i)} = \frac{18+6i}{9-i^2} = \frac{18+6i}{9-(-1)} = \frac{18+6i}{10} = \frac{9+3i}{5}$ (F, L)

$\frac{3i+1}{5} - \frac{9+3i}{5} = \frac{3i+1-9-3i}{5} = -\frac{8}{5}$

$\boxed{-\frac{8}{5} + 0i}$

6. Simplify and write the answer in standard form.

a) $i^{23} = i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^4 \cdot i^3$

$\sqrt{-1} = i$

$i^2 = -1$

$i^3 = -i$

$i^4 = 1$

$i^{23} = 1 \cdot 1 \cdot 1 \cdot 1 \cdot 1 \cdot -i$

$= -i$

$\boxed{0 - i}$

b) $7i^2 - 3i^{33} = 7(-1) - 3i^{32} \cdot i^1$

$= -7 - 3(1)(i)$

$= \boxed{-7 - 3i}$

c) $\left(\sqrt{-3}\right)^6$

$\sqrt{-1}\ \sqrt{3}$

$(\sqrt{3}i)^6$

$(\sqrt{3}i)(\sqrt{3}i)(\sqrt{3}i)(\sqrt{3}i)(\sqrt{3}i)(\sqrt{3}i)$

$3i^2 \quad 3i^2 \quad 3i^2$

$27i^6 = 27i^4 \cdot i^2$

$27(1)(-1) = -27$

$\boxed{-27 + 0i}$

d) $\frac{1}{(2i)^7} = \frac{1}{2^7 i^7} = \frac{1}{128i^7}$

$\frac{1}{128i^4 \cdot i^3} = \frac{1}{128(1)(-i)} = \frac{1}{-128i} \cdot \frac{i}{i}$

$= \frac{i}{-128i^2} = \frac{i}{-128(-1)} = \frac{i}{128}$

$= \boxed{0 + \frac{1}{128}i}$