Solving Quadratic and Rational Inequalities

Steps to Solve Quadratic Inequalities

Step 1: Change the inequality to an equation.

Step 2: Set the equation equal to zero and factor.

Step 3: Solve for the variable.

Step 4: Place the solutions on a number line and test values in each interval.

Directions: Solve the inequality. Write the solution in interval notation and graph the solution on a number line.

1. $x^2 + 2x - 3 < 0$ *

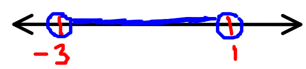
$x^2 + 2x - 3 = 0$

$(x+3)(x-1) = 0$

$x+3=0 \quad x-1=0$

$x=-3 \quad x=1$

$\boxed{(-3, 1)}$



$x^2 + 2x - 3 < 0$

$x=-4 \quad (-4)^2 + 2(-4) - 3$

$16 - 8 - 3 = 5$

$5 < 0$ NO

$x=0 \quad 0^2 + 2(0) - 3$

$0 + 0 - 3$

$-3 < 0$ YES

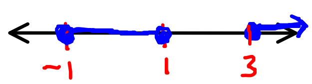
$x=2 \quad (2)^2 + 2(2) - 3$

$4 + 4 - 3$

$5 < 0$ NO

2. $x^3 - 3x^2 - x + 3 \geq 0$

$\boxed{[-1, 1] \cup [3, \infty)}$



$\frac{x^3}{x^2} - \frac{3x^2}{x^2} \Big| \frac{-x}{-1} + \frac{3}{-1} = 0$

GCF $= x^2$ · GCF $= -1$

$\frac{x^2(x-3)}{x-3} - \frac{1(x-3)}{x-3} = 0$

GCF $= (x-3)$

$(x-3)(x^2-1) = 0$

$(x-3)(x+1)(x-1) = 0$

$x-3=0 \quad x+1=0 \quad x-1=0$

$x=3 \quad x=-1 \quad x=1$

$x^3 - 3x^2 - x + 3 \geq 0$

$x=-2 \quad (-2)^3 - 3(-2)^2 - (-2) + 3$

$-8 - 12 + 2 + 3$

$-15 \geq 0$ NO

$x=0 \quad 0^3 - 3(0)^2 - 0 + 3$

$0 - 0 - 0 + 3$

$3 \geq 0$ YES

$x=2 \quad (2)^3 - 3(2)^2 - 2 + 3$

$8 - 12 - 2 + 3$

$-3 \geq 0$ NO

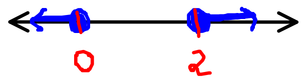
$x=4 \quad (4)^3 - 3(4)^2 - 4 + 3$

$64 - 48 - 4 + 3$

$15 \geq 0$ YES

3. $2x^3 - x^4 \le 0$

$\boxed{(-\infty, 0] \cup [2, \infty)}$



$2x^3 - x^4 = 0$

$x^3(2 - x) = 0$ GCF $= x^3$

$\sqrt[3]{x^3} = \sqrt[3]{0}$ $2 - x = 0$

$+x \quad +x$

$x = 0$ $x = 2$

$2x^3 - x^4 \le 0$

$x = -1$ $2(-1)^3 - (-1)^4$

$-2 - 1$

$-3 \le 0$ yes

$x = 1$ $2(1)^3 - (1)^4$

$2 - 1$

$1 \le 0$ NO

$x = 3$ $2(3)^3 - (3)^4$

$54 - 81$

$-27 \le 0$ yes

Steps to Solve Rational Inequalities

Step 1: Change the inequality to an equation.

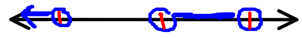
Step 2: Combine the fractions.

Step 3: Set the numerator and denominator equal to zero and solve for the variable.

Step 4: Place the solutions on a number line and test values in each interval.

4. $\frac{5}{x} - x > 4$ *

$\boxed{(-\infty, -5) \cup (0, 1)}$



$\frac{5}{x} - x = 4$

$\quad -4 \quad -4$

$\frac{5}{x} - \frac{x \cdot x}{x \cdot 1} - \frac{4 \cdot x}{1 \cdot x} = 0$

LCD = x

$\frac{5}{x} - \frac{x^2}{x} - \frac{4x}{x} = 0$

$\frac{-x^2 - 4x + 5}{x} = 0$

Num = 0 $\qquad$ Den = 0

$\frac{-x^2}{-1} - \frac{4x}{-1} + \frac{5}{-1} = \frac{0}{-1}$ $\qquad$ $x = 0$

$x^2 + 4x - 5 = 0$

$(x+5)(x-1) = 0$

$x + 5 = 0 \qquad x - 1 = 0$

$x = -5 \qquad x = 1$

$\frac{5}{x} - x > 4$

$x = -6$: $\frac{5}{-6} - (-6) = \frac{-5}{6} + 6 > 4$ yes

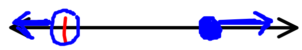
$x = -1$: $\frac{5}{-1} - (-1) = -5 + 1 = -4 > 4$ NO

$x = 1/2$: $\frac{5}{\frac{1}{2}} - \frac{1}{2} = 10 - \frac{1}{2} = 9\frac{1}{2} > 4$ yes

$x = 2$: $\frac{5}{2} - 2 = 2\frac{1}{2} - 2 = \frac{1}{2} > 4$ NO

5. $\frac{x+12}{x+2} - 3 \le 0$ *

$\boxed{(-\infty, -2) \cup [3, \infty)}$



$\frac{x+12}{x+2} - \frac{3 \cdot (x+2)}{1 \cdot (x+2)} = 0$

LCD = $x + 2$

$\frac{x+12}{x+2} - \frac{3(x+2)}{x+2} = 0$

$\frac{x + 12 - 3(x+2)}{x+2} = 0$

$\frac{x + 12 - 3x - 6}{x+2} = 0$

$\frac{-2x + 6}{x+2} = 0$

Num = 0 $\qquad$ den = 0

$-2x + 6 = 0 \qquad x + 2 = 0$

$\quad -6 \; -6 \qquad -2 \; -2$

$\frac{-2x}{-2} = \frac{-6}{-2} \qquad x = -2$

$x = 3$

$\frac{x+12}{x+2} - 3 \le 0$

$x = -3$: $\frac{-3+12}{-3+2} - 3 = \frac{9}{-1} - 3 = -9 - 3$

$-12 \le 0$ yes

$x = 0$: $\frac{0+12}{0+2} - 3 = \frac{12}{2} - 3 = 6 - 3 = 3$

$3 \le 0$ NO

$x = 4$: $\frac{4+12}{4+2} - 3 = \frac{16}{6} - 3 = \frac{8}{3} - 3$

$2\frac{2}{3} - 3 \le 0$

yes