Direct, Inverse and Joint Variation

y varies directly as x	y varies inversely as x	y varies jointly as x and z
$y = kx$	$y = \frac{k}{x}$	$y = kxz$
y increases, x increases y decreases, x decreases	y increases, x decreases y decreases, x increases	y varies as the product of two or more quantities

$k =$ constant of variation

1. x varies directly as y. Find x when $y = 24$ and $k = 3$.

$x = k \cdot y$

$x = 3 \cdot 24$

$\boxed{x = 72}$

2. A varies directly as the square of B. Find A when $B = 9$ and $k = \frac{3}{4}$.

$A = k \cdot B^2$

$A = \frac{3}{4} \cdot \frac{9^2}{1}$

$A = \frac{3}{4} \cdot \frac{81}{1}$

$\boxed{A = \frac{243}{4}}$

3. x varies inversely as y. Find x when $y = 30$ and $k = \frac{1}{2}$.

$x = \frac{K}{y}$

$x = \frac{\frac{1}{2}}{30}$ $\quad \frac{1}{2} \div \frac{30}{1} = \frac{1}{2} \cdot \frac{1}{30} = \frac{1}{60}$

$\boxed{x = \frac{1}{60}}$

4. x varies jointly as y and z. Find x when $y = 15$, $z = 3$ and $k = \frac{2}{3}$.

$x = K \cdot yz$

$x = \frac{2}{3} \cdot \frac{15}{1} \cdot \frac{3}{1}$

$\boxed{x = 30}$

5. A varies directly as B and inversely as C. Find A when $B = 5$, $C = 20$ and $k = 4$.

$A = \frac{K \cdot B}{C}$

$A = \frac{4 \cdot 5}{20}$

$\boxed{A = 1}$

6. U varies jointly as V and W and inversely as the square of x. Find U when $V=6,\ W=7,\ x=9$ and $k=\frac{3}{2}$.

$$U=\frac{k\cdot V\cdot W}{x^2}$$

$$U=\frac{\frac{3}{2}\cdot\frac{6}{1}\cdot\frac{7}{1}}{9^2}=\frac{63}{81}$$

$$\boxed{U=\frac{7}{9}}$$

7. x varies directly as y. If $x=12$ when $y=3$, find x when $y=5$.

$$*\ x=k\cdot y$$

$$\frac{12}{3}=\frac{k\cdot 3}{3}$$

$$k=4$$

$$x=k\cdot y$$

$$x=4\cdot 5$$

$$\boxed{x=20}$$

8. A varies inversely as the square of P. If $A=5$ when $P=5$, find A when $P=10$.

$*A = \frac{K}{P^2}$

$5 = \frac{K}{5^2}$

$\frac{5}{1} = \frac{K}{25}$

$K = 125$

$A = \frac{K}{P^2}$

$A = \frac{125}{10^2} = \frac{125^5}{100_4}$

$\boxed{A = \frac{5}{4}}$

9. F varies jointly as M_1 and M_2 and inversely as d. If $F=30$ when $M_1=6$, $M_2=10$ and $d=\frac{2}{5}$, find F when $M_1=12$, $M_2=20$ and $d=\frac{4}{5}$.

$*F = \frac{K \cdot M_1 \cdot M_2}{d}$

$30 = \frac{K \cdot 6 \cdot 10}{\frac{2}{5}}$

$\frac{30}{1} = \frac{60K}{\frac{2}{5}}$

$60K = 30 \cdot \frac{2}{5}$

$\frac{60K}{60} = \frac{12}{60}$

$K = \frac{1}{5}$

$F = \frac{K \cdot M_1 \cdot M_2}{d}$

$F = \frac{\frac{1}{5} \cdot 12 \cdot 20}{\frac{4}{5}}$

$F = 48 \div \frac{4}{5} = 48 \cdot \frac{5}{4}$

$\boxed{F = 60}$

10. The recommended dosage of a certain medication is directly proportional to a person's weight. If Laurie weighs 125 pounds and is given 2500 milligrams, find the recommended dosage for Larry who weighs 165 pounds.

d = dosage
w = weight

* $d = kw$

$\frac{2500}{125} = \frac{k \cdot 125}{125}$

$k = 20$

Laurie
$w = 125$
$d = 2500$

Larry
$w = 165$
$d =$

$d = k \cdot w$
$d = 20 \cdot 165$

$d = 3300$ mg

11. If it takes 8 hours for 6 painters to paint a house, how long will it take 5 painters to paint a house of the same size?

h = hours
p = painters

* inversely related

* $h = \frac{k}{p}$

$h = 8$
$p = 6$

$\frac{8}{1} = \frac{k}{6}$

$k = 48$

$p = 5$
$h =$

$h = \frac{k}{p}$

$h = \frac{48}{5} = 9\frac{3}{5}$ hours

12. The intensity of light, I, varies inversely as the square of the distance, d. If the light intensity is 300-foot candles at 25 feet, find the light intensity at 15 feet.

* $I = \frac{K}{d^2}$

$I = 300$

$d = 25$

$300 = \frac{K}{25^2}$

$\frac{300}{1} = \frac{K}{625}$

$K = 187{,}500$

$I =$

$d = 15$

$I = \frac{187{,}500}{15^2}$

$I = \frac{187{,}500}{225}$

$I = 833.33$ - foot candles