Conic Sections -
Hyperbolas and
Classifying Conics

Standard Form for the Equation of a Hyperbola

Horizontal Hyperbola

$$\frac{(x-h)^2}{a^2}-\frac{(y-k)^2}{b^2}=1$$

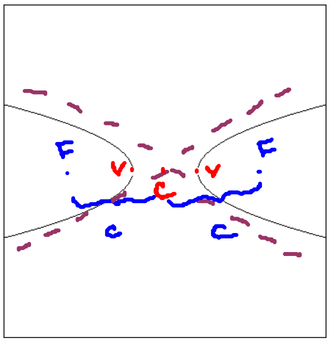
- Asymptotes: $y-k=\pm\frac{b}{a}(x-h)$

$y - y_1 = m(x - x_1)$

Vertical Hyperbola

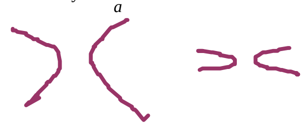
$$\frac{(y-k)^2}{a^2}-\frac{(x-h)^2}{b^2}=1$$

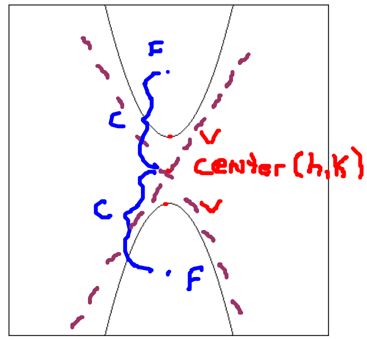
Asymptotes: $y-k=\pm\frac{a}{b}(x-h)$



a is always in the denominator of the positive fraction

- Center $=(h,k)$
- Vertices: a units from the center
- Foci: $c^2=a^2+b^2$
- Eccentricity: $e=\frac{c}{a}$





1. Find the center, vertices, foci, eccentricity and equations of the asymptotes of the hyperbola and sketch its graph.

a) $\frac{(x-1)^2}{9}-\frac{y^2}{25}=1$

Horizontal

$\boxed{\text{center } (1,0)}$

$a^2=9 \quad a=3$

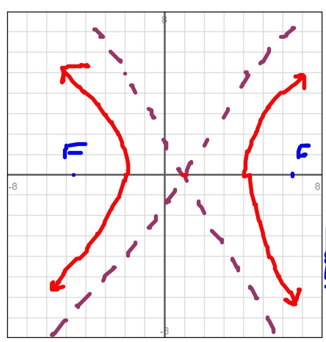
$b^2=25 \quad b=5$

$y-k=\pm\frac{b}{a}(x-h)$

$y-0=\pm\frac{5}{3}(x-1)$

$\boxed{y=\pm\frac{5}{3}(x-1)}$

$\boxed{V: (4,0) \quad (-2,0)}$



$e=\frac{c}{a}$

$e=\frac{\sqrt{34}}{3}$

$\boxed{e\approx 1.9}$

F: $c^2=a^2+b^2$

$c^2=9+25$

$c^2=34$

$c=\sqrt{34}\approx 5.8$

$\boxed{(1+\sqrt{34},0) \quad (1-\sqrt{34},0)}$

b) $-4x^2+24x+y^2+4y-41=0$ $\quad +41$

$-4x^2+24x+y^2+4y=41$

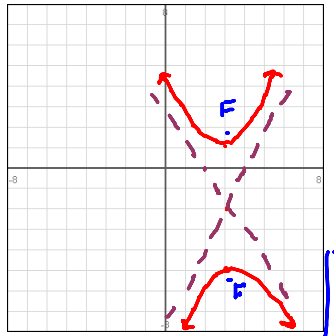
$-4(x^2-6x+9)+(y^2+4y+4)=41$ $\quad -36 \quad +4$

$\frac{6}{2}=(3)^2=9 \qquad \frac{4}{2}=(2)^2=4$

$\frac{-4(x-3)^2}{9}+\frac{(y+2)^2}{9}=\frac{9}{9}$

$-\frac{(x-3)^2}{\frac{9}{4}}+\frac{(y+2)^2}{9}=1$

$\frac{(y+2)^2}{9}-\frac{(x-3)^2}{9/4}=1$



$F: c^2=a^2+b^2$

$c^2=9+\frac{9}{4}$

$c^2=\frac{45}{4}$

$c=\frac{\sqrt{45}}{\sqrt{4}}=\frac{3\sqrt{5}}{2}$

$\boxed{F: \left(3, -2\pm\frac{3\sqrt{5}}{2}\right)}$

$\boxed{\text{center: } (3,-2)}$

$a^2=9 \qquad a=3$

$b^2=9/4 \qquad b=3/2$

$\boxed{V: (3,1)(3,-5)}$

$y-k=\pm\frac{a}{b}(x-h)$

$y+2=\pm\frac{3}{\frac{3}{2}}(x-3)$

$\boxed{y+2=\pm 2(x-3)}$

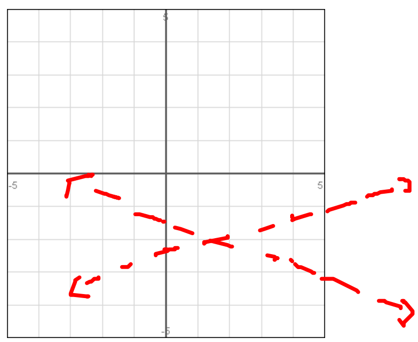
$e=\frac{c}{a}=\frac{\frac{3\sqrt{5}}{2}}{3}=\frac{\cancel{3}\sqrt{5}}{2}\cdot\frac{1}{\cancel{3}}=\boxed{\frac{\sqrt{5}}{2}}$

c) $16y^2-x^2+2x+64y+63=0$ $\quad -63$

$-x^2+2x+16y^2+64y=-63$

$-1(x^2-2x+1)+16(y^2+4y+4)=-63$ $\quad -1 \quad +64$

$\frac{2}{2}=(1)^2=1 \qquad \frac{4}{2}=(2)^2=4$



$-(x-1)^2+16(y+2)^2=0$

$+(x-1)^2 \qquad +(x-1)^2$

$\sqrt{16(y+2)^2}=\sqrt{(x-1)^2}$

$4(y+2)=\pm(x-1)$

$4(y+2)=(x-1)$

$4y+8=x-1$

$-8 \qquad -8$

$\frac{4y}{4}=\frac{x}{4}-\frac{9}{4}$

$y=\frac{1}{4}x-\frac{9}{4}$

$4(y+2)=-(x-1)$

$4y+8=-x+1$

$-8 \qquad -8$

$\frac{4y}{4}=\frac{-x}{4}-\frac{7}{4}$

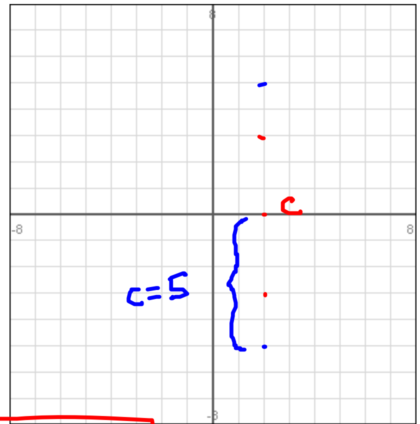
$y=-\frac{1}{4}x-\frac{7}{4}$

2. Find the standard form of the equation of the hyperbola.

a) Vertices: $(2,3)$ and $(2,-3)$

Foci: $(2,5)$ and $(2,-5)$

center $\boxed{(2,0)}$ vertical



* $\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$\boxed{a^2=9}$

$a=3 \qquad c=5$

$c^2=a^2+b^2$

$25=9+b^2$

$-9 \quad -9$

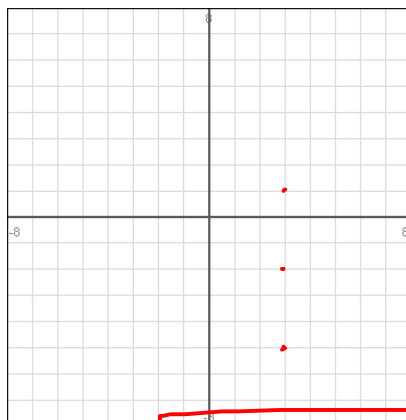
$\boxed{b^2=16}$

$$\boxed{\frac{y^2}{9}-\frac{(x-2)^2}{16}=1}$$

b) Vertices: $(3,-5)$ and $(3,1)$

Asymptotes: $y = 2x - 8$ and $y = -2x + 4$

center $(3,-2)$



$\frac{(y-k)^2}{a^2} - \frac{(x-h)^2}{b^2} = 1$

$a=3 \qquad a^2=9$

$\frac{a}{b} = \frac{2}{1} = \frac{3}{b}$

$\frac{2b}{2}=\frac{3}{2} \qquad b=\frac{3}{2} \qquad b^2=\frac{9}{4}$

$$\boxed{\frac{(y+2)^2}{9}-\frac{(x-3)^2}{9/4}=1}$$

c) Vertices: $(-2,1)$ and $(2,1)$

Passes through the point: $(8,4)$

$x \quad y$

center $(0,1)$

$$\frac{(x-h)^2}{a^2} - \frac{(y-k)^2}{b^2} = 1$$

$a=2 \quad a^2=4$

$$\frac{(8-0)^2}{4} - \frac{(4-1)^2}{b^2} = 1$$

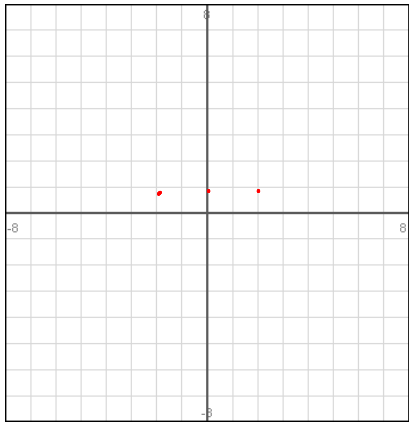
$$16 - \frac{9}{b^2} = 1$$

$-16 \qquad -16$

$$-\frac{9}{b^2} = \frac{-15}{1}$$

$$\frac{-15b^2}{-15} = \frac{-9}{-15}$$

$$b^2 = 9/15$$



$$\boxed{\frac{x^2}{4} - \frac{(y-1)^2}{9/15} = 1}$$

Classifying Conics from General Equations

Circle - x^2 and y^2 have the same coefficients

Parabola - only x^2 or y^2

Ellipse - x^2 and y^2 have different coefficients but are the same sign

Hyperbola - x^2 and y^2 have different signs

3. Classify each of the following conic sections.

a) $4x^2 - y^2 - 4x + 3 = 0$

$\boxed{\text{Hyperbola}}$

b) $4x^2 + 3y^2 - 14x + 3y = -7$

$\boxed{\text{Ellipse}}$

c) $-2x^2 - 2y^2 - 16x + 15 = 0$

$\boxed{\text{Circle}}$

d) $y^2 - 4y + x = 0$

$\boxed{\text{Parabola}}$