Graphs of Logarithmic Functions

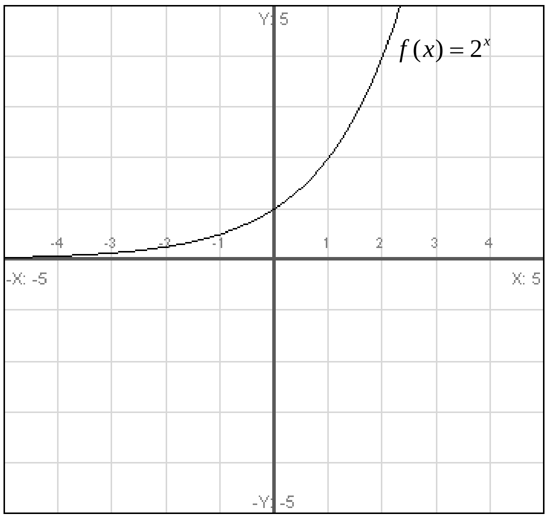
$y = \log_a x$ represents the graph of a logarithmic function where $x > 0$, $a > 0$ and $a \neq 1$.

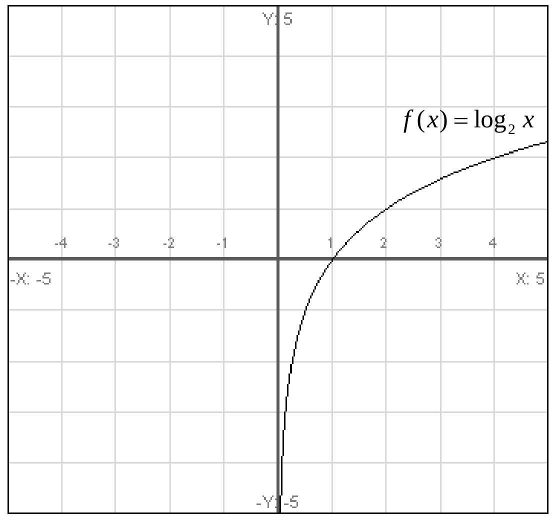
Exponential Function

$f(x) = a^x$

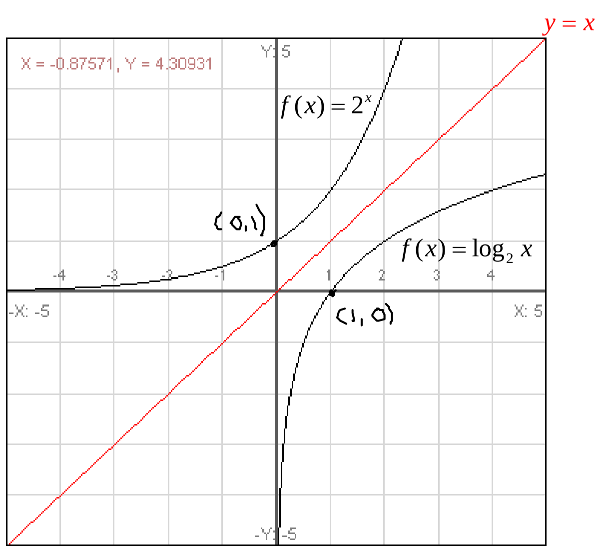
Logarithmic Function

$f(x) = \log_a x$





The logarithmic and exponential functions are inverses. Therefore, they are symmetrical about the line *y = x*.



Properties of the Exponential Function

Domain: All Real Numbers
Range: *y > 0*
y-intercept: *(0, 1)*
Horizontal Asymptote: *y = 0*
The function is increasing.

Properties of the Logarithmic Function

Domain: *x > 0*
Range: All Real Numbers
x-intercept: *(1, 0)*
Vertical Asymptote: *x = 0*
The function is increasing.

Properties of $y = \log_2 x$

Domain: $x > 0$

Range: All real numbers

x-intercept: $(1, 0)$

Vertical Asymptote: $x = 0$

The function is increasing.

Properties of $y = \log_4 x$

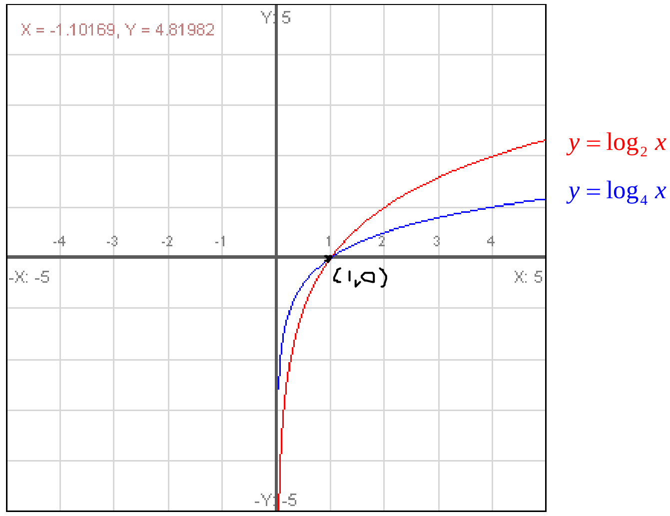
Domain: $x > 0$

Range: All real numbers

x-intercept: $(1, 0)$

Vertical Asymptote: $x = 0$

The function is increasing.

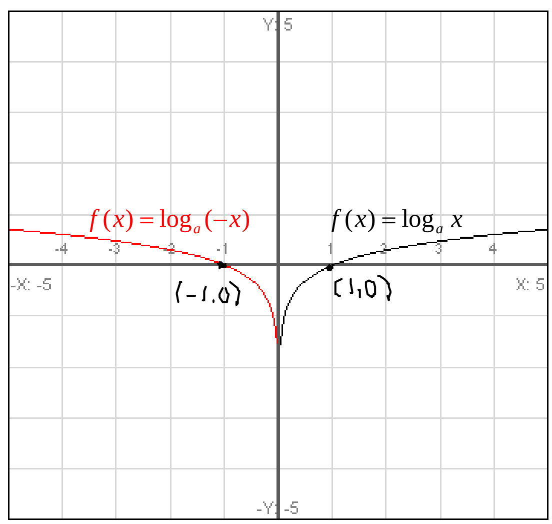


The graph of $y = \log_4 x$ increases more slowly and is closer to the vertical asymptote.

Transformations of the Graph of the Logarithmic Function

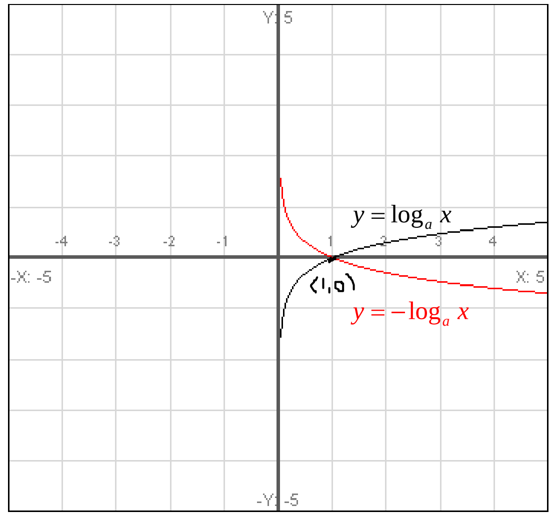
$y = \log_a(-x)$

Reflected over the vertical asymptote
Domain: *x < 0*
Range: All Real Numbers
x-intercept: *(-1, 0)*
Vertical Asymptote: *x = 0*
The function is decreasing.



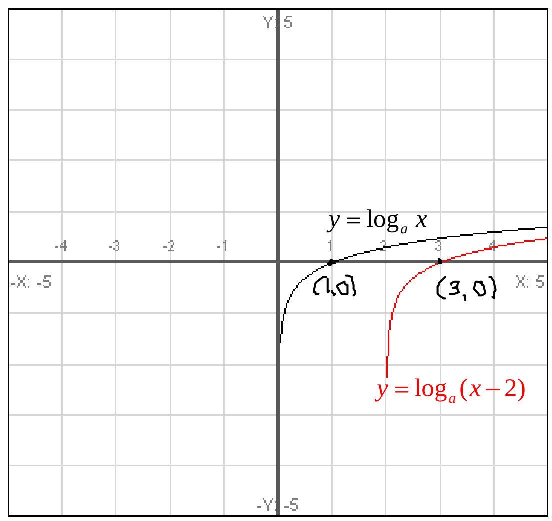
$y = -\log_a x$

Reflected over the known point
Domain: *x > 0*
Range: All Real Numbers
x-intercept: *(1, 0)*
Vertical Asymptote: *x = 0*
The function is decreasing.



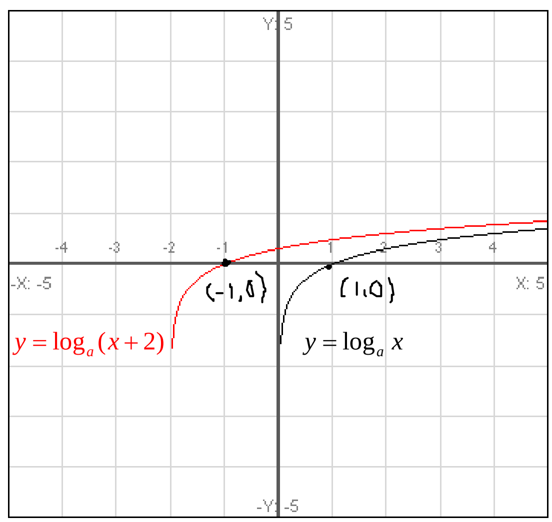
$y = \log_a (x - h)$

Shifted *h* units to the right
Domain: *x > h* x > 2
Range: All Real Numbers
x-intercept: *(1 + h, 0)* (3, 0)
Vertical Asymptote: *x = h* x = 2
The function is increasing.



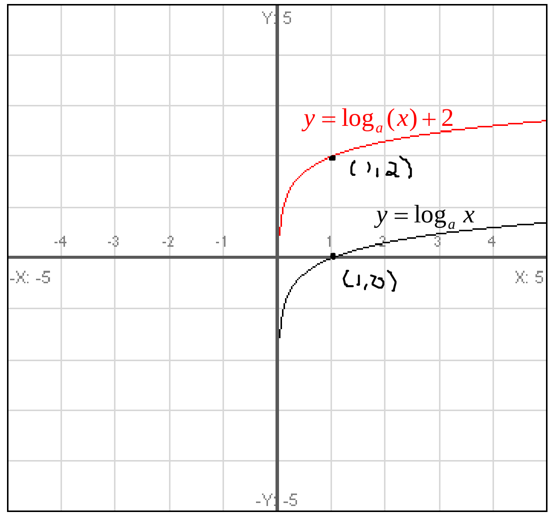
$y = \log_a (x + h)$

Shifted *h* units to the left
Domain: *x > h* x > -2
Range: All Real Numbers
x-intercept: *(1 + h, 0)* (-1, 0)
Vertical Asymptote: *x = h* x = -2
The function is increasing.



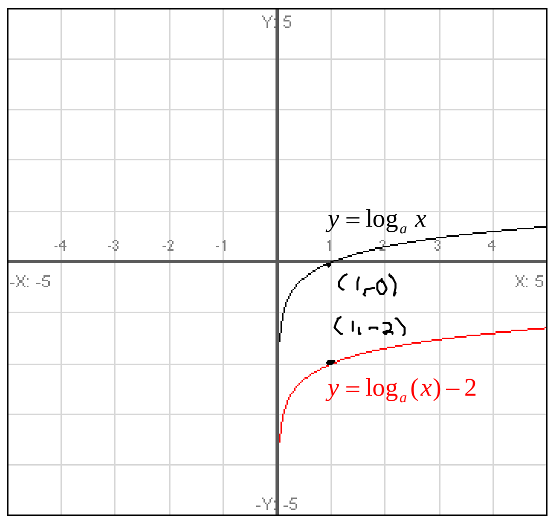
$y = \log_a(x) + k$

Shifted *k* units up
Domain: *x > 0*
Range: All Real Numbers
Vertical Asymptote: *x = 0*
The function is increasing.



$y = \log_a(x) - k$

Shifted *k* units down
Domain: *x > 0*
Range: All Real Numbers
Vertical Asymptote: *x = 0*
The function is increasing.



Directions: Graph each logarithmic function. Identify the domain, range, asymptotes, intercepts and determine if the function is increasing or decreasing.

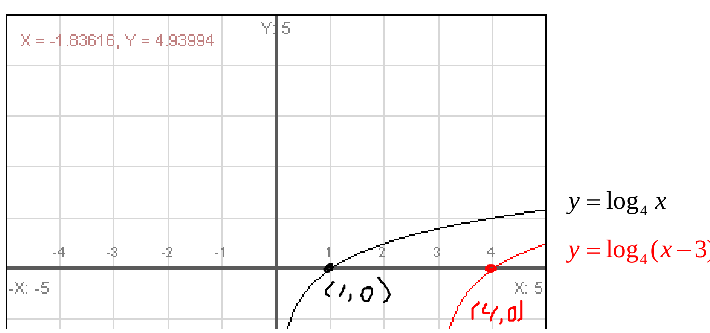
1. $y = \log_4(x-3)$

3 units to the right

Domain: $x > 3$

Range: All real numbers

Asymptote: $x = 3$



1. $y=\log_4(x-3)$

3 units to the right

Domain: $x > 3$

Range: All real numbers

Asymptote: $x = 3$

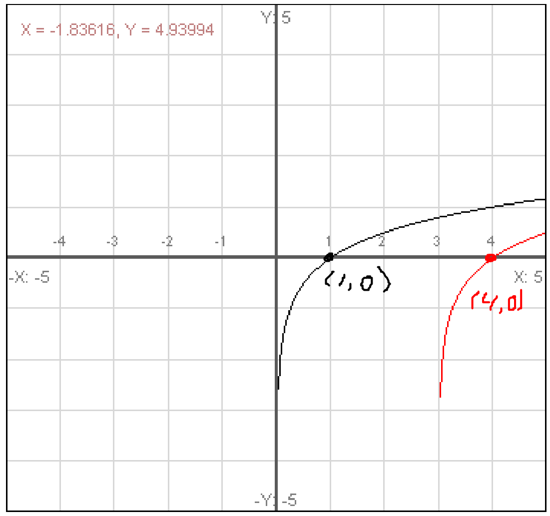
Intercept: $0 = \log_4(x-3)$

set y=0 $4^0 = x-3$

$1 = x-3$

$x = 4$ $(4,0)$

Increasing/Decreasing: ↑



$y=\log_4 x$

$y=\log_4(x-3)$

2. $y=-\log_4(x-2)$

2 units right

Reflected around known point

Domain: $x > 2$

Range: All real numbers

Asymptote: $x = 2$

Intercept: $\frac{0}{-1} = \frac{-\log_4(x-2)}{-1}$

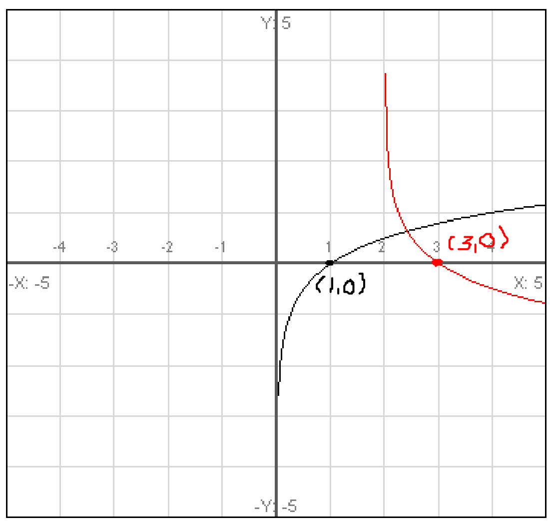
$0 = \log_4(x-2)$

$4^0 = x-2$

$1 = x-2$

$x = 3$ $(3,0)$

Increasing/Decreasing: ↓



$y=\log_4 x$

$y=-\log_4(x-2)$

3. $y = -3 - \log_2(-x+4)$

$y = -\log_2(-x+4) - 3$

$-x+4=0$ 4 units right, 3 units down

$x = 4$

Reflected over vertical asymptote and known point

Domain: $x < 4$

Range: All real numbers

Asymptote: $x = 4$

Intercept: $0 = -3 - \log_2(-x+4)$

$+3 \quad +3$

$\frac{3}{-1} = \frac{-\log_2(-x+4)}{-1}$

$-3 = \log_2(-x+4)$

$2^{-3} = -x+4$

$\frac{1}{2^3} = -x+4$

$\frac{1}{8} = -x+4$

$-4 \quad -4$

$-x = -31/8$ $(31/8, 0)$

Increasing/Decreasing: ↑