

Properties of Logarithms - Part 2

Logarithmic Form

$$\log_a x = y$$

$$a^y = x$$

Exponential Form

$$x = a^y$$

Properties of Logarithms

$$1. \log_a 1 = 0$$

$$2. \log_a a = 1$$

$$3. \log_a a^x = x$$

$$4. \text{ If } \log_a x = \log_a y, \text{ then } x = y.$$

$$5. \log_a (u \cdot v) = \log_a u + \log_a v$$

$$6. \log_a \frac{u}{v} = \log_a u - \log_a v$$

$$7. \log_a u^n = n \cdot \log_a u$$

$$8. \log_a \sqrt[n]{u} = \frac{1}{n} \cdot \log_a u \quad \log_a u^{\frac{1}{n}} = \frac{1}{n} \log_a u$$

$$9. \log_a x = \frac{\log_{10} x}{\log_{10} a} \text{ where } a \neq 10 \text{ (Change-of-Base Formula)}$$

Directions: Use the properties of logarithms to expand each logarithmic expression.

$$1. \log_{10} 6x = \boxed{\log_{10} 6 + \log_{10} x}$$

$$2. \log_6 y^4 = \boxed{4 \log_6 y}$$

$$\begin{aligned}
 3. \log x(\sqrt{x-1}) &= \log x + \log \sqrt{x-1} \\
 &= \log x + \overbrace{\log(x-1)^{\frac{1}{2}}}^{\text{cancel}} \\
 &= \boxed{\log x + \frac{1}{2} \log(x-1)}
 \end{aligned}$$

$$4. \log_7 \frac{x}{y} = \boxed{\log_7 x - \log_7 y}$$

$$\begin{aligned}
 5. \log x\sqrt{y} &= \log x + \log \sqrt{y} \\
 &= \log x + \overbrace{\log y^{\frac{1}{2}}}^{\text{cancel}} \\
 &= \boxed{\log x + \frac{1}{2} \log y}
 \end{aligned}$$

$$\begin{aligned}
 6. \log \frac{x^2}{y^3 z^4} &= \overbrace{\log x^2}^{\text{cancel}} - \overbrace{\log y^3}^{\text{cancel}} - \overbrace{\log z^4}^{\text{cancel}} \\
 &= \boxed{2 \log x - 3 \log y - 4 \log z}
 \end{aligned}$$

$$\begin{aligned}
 7. \log(a^2 b^3 c^4) &= \log a^8 b^{12} c^4 = \overbrace{\log a^8}^{\text{cancel}} + \overbrace{\log b^{12}}^{\text{cancel}} + \overbrace{\log c^4}^{\text{cancel}} \\
 &= \boxed{8 \log a + 12 \log b + 4 \log c}
 \end{aligned}$$

$$\begin{aligned}
 8. \log \sqrt{5a^6 b^7} &= \log (5a^6 b^7)^{\frac{1}{2}} = \log 5^{\frac{1}{2}} a^3 b^{\frac{7}{2}} \\
 &= \overbrace{\log 5^{\frac{1}{2}}}^{\text{cancel}} + \overbrace{\log a^3}^{\text{cancel}} + \overbrace{\log b^{\frac{7}{2}}}^{\text{cancel}} \\
 &= \boxed{\frac{1}{2} \log 5 + 3 \log a + \frac{7}{2} \log b}
 \end{aligned}$$

$$\begin{aligned}
 9. \log_4 \sqrt[4]{\frac{x-6}{3y^2}} &= \log_4 \left(\frac{x-6}{3y^2} \right)^{\frac{1}{4}} = \frac{1}{4} \log_4 \frac{x-6}{3y^2} \\
 &= \frac{1}{4} \left[\log_4(x-6) - \log_4 3 - \log_4 y^2 \right] \\
 &= \frac{1}{4} [\log_4(x-6) - \log_4 3 - 2 \log_4 y] \\
 &= \boxed{\frac{1}{4} \log_4(x-6) - \frac{1}{4} \log_4 3 - \frac{1}{2} \log_4 y}
 \end{aligned}$$

Directions: Write each expression as a single logarithm.

$$10. \log x + \log 7 = \boxed{\log 7x}$$

$$\begin{aligned}
 11. \log x - 3 \log y &= \log x - \log y^3 \\
 &= \boxed{\log \frac{x}{y^3}}
 \end{aligned}$$

$$\begin{aligned}
 12. -3 \log(x-5) + \frac{1}{3} \log 4x &= \log(x-5)^{-3} + \log(4x)^{\frac{1}{3}} = \log \frac{1}{(x-5)^3} + \log \sqrt[3]{4x} \\
 &= \log \frac{1}{(x-5)^3} \cdot \sqrt[3]{4x} = \boxed{\log \frac{\sqrt[3]{4x}}{(x-5)^3}}
 \end{aligned}$$

$$\begin{aligned}
 13. 4 \log_7 2 + 5 \log_7 x - 6 \log_7 z &= \log_7 2^4 + \log_7 x^5 - \log_7 z^6 \\
 &= \log_7 16 + \log_7 x^5 - \log_7 z^6 \\
 &= \boxed{\log_7 \frac{16 \cdot x^5}{z^6}}
 \end{aligned}$$

$$14. \frac{1}{3}(2 \log(x+2) - 6 \log(x-1) - 3 \log x) = \frac{2}{3} \log(x+2) - 2 \log(x-1) - \log x$$

$$\begin{aligned}
 &= \log(x+2)^{\frac{2}{3}} - \log(x-1)^2 - \log x \\
 &= \log \sqrt[3]{(x+2)^2} - \log(x-1)^2 - \log x \\
 &= \boxed{\log \frac{\sqrt[3]{(x+2)^2}}{(x-1)^2 \cdot x}}
 \end{aligned}$$