Properties of Logarithms - Part 2

Logarithmic Form	Exponential Form
$\log_a x = y$	$x = a^y$

$a^y = x$

Properties of Logarithms

1. $\log_a 1 = 0$
2. $\log_a a = 1$
3. $\log_a a^x = x$
4. If $\log_a x = \log_a y$, then $x = y$.
5. $\log_a (u \bullet v) = \log_a u + \log_a v$
6. $\log_a \frac{u}{v} = \log_a u - \log_a v$
7. $\log_a u^n = n \bullet \log_a u$
8. $\log_a \sqrt[n]{u} = \frac{1}{n} \bullet \log_a u$ $\log_a u^{\frac{1}{n}} = \frac{1}{n} \log_a u$
9. $\log_a x = \frac{\log_{10} x}{\log_{10} a}$ where $a \neq 10$ (Change-of-Base Formula)

Directions: Use the properties of logarithms to expand each logarithmic expression.

1. $\log_{10} 6x = \boxed{\log_{10} 6 + \log_{10} x}$

2. $\log_6 y^4 = \boxed{4 \log_6 y}$

3. $\log x\left(\sqrt{x-1}\right) = \log x + \log\sqrt{x-1}$

$= \log x + \log (x-1)^{\frac{1}{2}}$

$= \boxed{\log x + \frac{1}{2}\log(x-1)}$

4. $\log_7 \dfrac{x}{y} = \boxed{\log_7 x - \log_7 y}$

5. $\log x\sqrt{y} = \log x + \log\sqrt{y}$

$= \log x + \log y^{\frac{1}{2}}$

$= \boxed{\log x + \frac{1}{2}\log y}$

6. $\log \dfrac{x^2}{y^3 z^4} = \log x^2 - \log y^3 - \log z^4$

$= \boxed{2\log x - 3\log y - 4\log z}$

7. $\log\left(a^2 b^3 c\right)^4 \qquad \log a^8 b^{12} c^4 = \log a^8 + \log b^{12} + \log c^4$

$= \boxed{8\log a + 12\log b + 4\log c}$

8. $\log\sqrt{5a^6b^7} = \log\left(5a^6b^7\right)^{\frac{1}{2}} = \log 5^{\frac{1}{2}} a^3 b^{\frac{7}{2}}$

$= \log 5^{\frac{1}{2}} + \log a^3 + \log b^{\frac{7}{2}}$

$= \boxed{\frac{1}{2}\log 5 + 3\log a + \frac{7}{2}\log b}$

9. $\log \sqrt[4]{\dfrac{x-6}{3y^2}} = \log\left(\dfrac{x-6}{3y^2}\right)^{\frac{1}{4}} = \dfrac{1}{4}\log \dfrac{x-6}{3y^2}$

$= \dfrac{1}{4}\left[\log(x-6) - \log 3 - \log y^2\right]$

$= \dfrac{1}{4}\left[\log(x-6) - \log 3 - 2\log y\right]$

$= \boxed{\dfrac{1}{4}\log(x-6) - \dfrac{1}{4}\log 3 - \dfrac{1}{2}\log y}$

Directions: Write each expression as a single logarithm.

10. $\log x + \log 7 = \boxed{\log 7x}$

11. $\log x - 3\log y = \log x - \log y^3$

$= \boxed{\log \dfrac{x}{y^3}}$

12. $-3\log(x-5) + \dfrac{1}{3}\log 4x = \log(x-5)^{-3} + \log(4x)^{\frac{1}{3}} = \log \dfrac{1}{(x-5)^3} + \log \sqrt[3]{4x}$

$= \log \dfrac{1}{(x-5)^3} \cdot \sqrt[3]{4x} = \boxed{\log \dfrac{\sqrt[3]{4x}}{(x-5)^3}}$

13. $4\log_7 2 + 5\log_7 x - 6\log_7 z = \log_7 2^4 + \log_7 x^5 - \log_7 z^6$

$= \log_7 16 + \log_7 x^5 - \log_7 z^6$

$= \boxed{\log \dfrac{16 \cdot x^5}{z^6}}$

14. $\dfrac{1}{3}\left(2\log(x+2) - 6\log(x-1) - 3\log x\right) = \dfrac{2}{3}\log(x+2) - 2\log(x-1) - \log x$

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$$= \log (x+2)^{\frac{2}{3}} - \log (x-1)^2 - \log x$$

$$= \log \sqrt[3]{(x+2)^2} - \log (x-1)^2 - \log x$$

$$= \boxed{\log \frac{\sqrt[3]{(x+2)^2}}{(x-1)^2 \cdot x}}$$