

Solving Exponential and Logarithmic Equations

Directions: Solve each exponential equation.

$$1. 2^{x+2} = 16^{5x}$$

$$2^{x+2} = (2^4)^{5x}$$

$$2^{x+2} = 2^{20x}$$

$$\begin{array}{r} x+2 \\ -x \end{array} = \begin{array}{r} 20x \\ -x \end{array}$$

$$\frac{2}{19} = \frac{19x}{19}$$

$$\boxed{x = \frac{2}{19}}$$

$$2. 27^{x+1} = 81^{x+5}$$

$$(3^3)^{x+1} = (3^4)^{x+5}$$

$$3^{3(x+1)} = 3^{4(x+5)}$$

$$3(x+1) = 4(x+5)$$

$$\begin{array}{r} 3x+3 \\ -3x \end{array} = \begin{array}{r} 4x+20 \\ -3x \end{array}$$

$$3 = x + 20$$

$$\begin{array}{r} -20 \\ -20 \end{array}$$

$$\boxed{x = -17}$$

$$3. \left(\frac{1}{5}\right)^x = 125^{x^2}$$

$$(5^{-1})^x = (5^3)^{x^2}$$

$$5^{-x} = 5^{3x^2}$$

$$\begin{array}{r} -x \\ +x \end{array} = \begin{array}{r} 3x^2 \\ +x \end{array}$$

$$0 = 3x^2 + x$$

$$x(3x+1) = 0$$

$$\boxed{x=0} \quad \begin{array}{l} 3x+1=0 \\ -1 \quad -1 \\ 3x=-1 \\ \boxed{x = -\frac{1}{3}} \end{array}$$

$$4. 2^{4-x} = 1$$

$$2^{4-x} = 2^0$$

$$4-x = 0$$

$$\boxed{x = 4}$$

Directions: Solve each exponential equation. Round your answer to the nearest thousandth.

$$5. 2^{5x} = 3000$$

$$\log 2^{5x} = \log 3000$$

$$5x \log 2 = \log 3000$$

$$\frac{x \cdot 5 \log 2}{5 \log 2} = \frac{\log 3000}{5 \log 2}$$

$$x = \frac{\log 3000}{5 \log 2} = \boxed{2.310}$$

$$\log 3000 \div (5 \times \log 2) =$$

$$6. 8^{x-2} = 250$$

$$6. \quad 8^{x-2} = 250$$

$$\log 8^{x-2} = \log 250$$

$$(x-2) \log 8 = \log 250$$

$$x-2 = \frac{\log 250}{\log 8} + 2$$

$$x = \frac{\log 250}{\log 8} + 2 = \boxed{4.655}$$

$$\log 250 = \div \log 8 = +2$$

$$7. \quad 14^{3x-4} = 10^{x-1}$$

$$\log 14^{3x-4} = \log 10^{x-1}$$

$$(3x-4) \log 14 = (x-1) \log 10 = 1$$

$$3x \log 14 - 4 \log 14 = x - 1$$

$$3x \log 14 - x = -1 + 4 \log 14$$

$$x (3 \log 14 - 1) = -1 + 4 \log 14$$

$$x = \frac{-1 + 4 \log 14}{3 \log 14 - 1} = \boxed{1.470}$$

$$-1 + 4 \times \log 14 = \div (3 \times \log 14 - 1) =$$

Directions: Solve each logarithmic equation. Round your answer to the nearest thousandth.

$$8. \quad \log_{10}(x-2) = 3$$

$$10^3 = x-2$$

$$1000 = x-2$$

$$+2 \quad +2$$

$$\boxed{x = 1002}$$

$$9. \log_4 x - \log_4 (x-2) = \frac{2}{3}$$

$$\log_4 \frac{x}{x-2} = \frac{2}{3}$$

$$\frac{4^{\frac{2}{3}}}{1} = \frac{x}{x-2}$$

$$x = 4^{2/3} (x-2)$$

$$x = 4^{2/3} x - 2 \cdot 4^{2/3}$$

$$-4^{2/3} x - 4^{2/3} x$$

$$x - 4^{2/3} x = 2 \cdot 4^{2/3}$$

$$x(1 - 4^{2/3}) = 2 \cdot 4^{2/3}$$

$$x = \frac{2 \cdot 4^{2/3}}{1 - 4^{2/3}} = -3.316$$

$$\square \quad \square y^x$$

$$4^{\square} (2 \div 3) = x 2 = \div (1 - 4^{\square} (2 \div 3)) =$$

$$10. \log_6 x + \log_6 (x-8) = \log_6 20$$

$$\log_6 x(x-8) = \log_6 20$$

$$x(x-8) = 20$$

$$x^2 - 8x = 20$$

$$-20 - 20$$

$$x^2 - 8x - 20 = 0$$

$$(x-10)(x+2) = 0$$

$$x-10=0 \quad x+2=0$$

$$\boxed{x=10} \quad x=-2$$