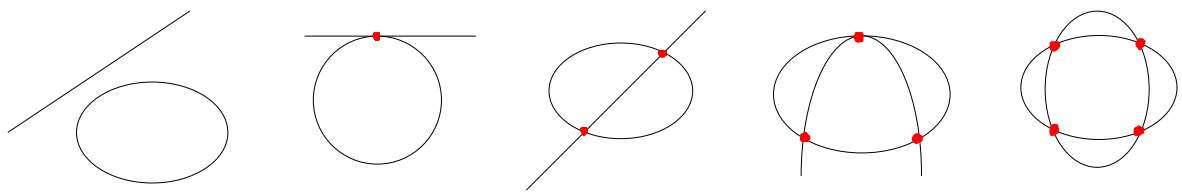
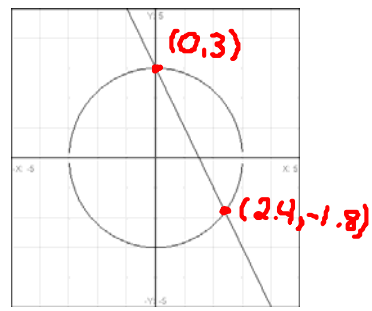
Solving Nonlinear Systems of Equations



No Solution | One Solution | Two Solutions | Three Solutions | Four Solutions

Directions: Solve the system of equations.

1. $x^2 + y^2 = 9$
   $2x + y = 3$



$2x + y = 3$

$y = \underbrace{3 - 2x}$

$x^2 + y^2 = 9$

$x^2 + (3-2x)^2 = 9$

$x^2 + (3-2x)(3-2x) = 9$

$x^2 + 9 - 6x - 6x + 4x^2 = 9$

$5x^2 - 12x = 0$

$x(5x - 12) = 0$

$x = 0 \qquad 5x - 12 = 0$

$5x = 12$

$x = 12/5$ OR 2.4

Substitute $x=0$ and $x=2.4$ into $y = 3-2x$

$x = 0$
$y = 3 - 2x$
$y = 3 - 2(0)$
$y = 3 - 0$
$y = 3$

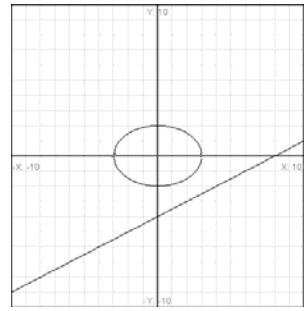
$\boxed{(0,3)}$

$x = 2.4$
$y = 3 - 2x$
$y = 3 - 2(2.4)$
$y = 3 - 4.8$
$y = -1.8$

$\boxed{(2.4, -1.8)}$

2. $4x^2 + 9y^2 = 36$

$x - 2y = 8$



$x - 2y = 8$

$x = 2y + 8$

$4x^2 + 9y^2 = 36$

$4(2y+8)^2 + 9y^2 = 36$

$4(2y+8)(2y+8) + 9y^2 = 36$

$4(4y^2 + 16y + 16y + 64) + 9y^2 = 36$

$4(4y^2 + 32y + 64) + 9y^2 = 36$

$16y^2 + 128y + 256 + 9y^2 = 36$

$25y^2 + 128y + 220 = 0$

$a = 25 \quad b = 128 \quad c = 220$

$$y = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

use the Quadratic Formula to solve for y

$$y = \frac{-128 \pm \sqrt{128^2 - 4(25)(220)}}{2(25)}$$

$$y = \frac{-128 \pm \sqrt{-5616}}{50}$$

Negative under the square root, therefore no solution

$\boxed{\text{NO solution}}$

3. $x^2 + y^2 = 4$

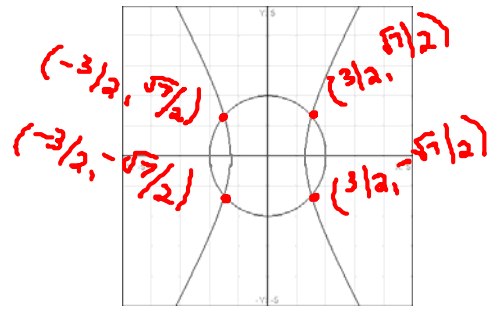
$3x^2 - y^2 = 5$

$$\begin{array}{r} x^2 + y^2 = 4 \\ 3x^2 - y^2 = 5 \\ \hline 4x^2 = 9 \end{array}$$

$4x^2 = 9$

$\sqrt{x^2} = \sqrt{\frac{9}{4}}$

$x = \pm \frac{3}{2}$



Substitute $x = \frac{3}{2}$ and $x = -\frac{3}{2}$ into $x^2 + y^2 = 4$

$x = \frac{3}{2}$

$x^2 + y^2 = 4$

$\left(\frac{3}{2}\right)^2 + y^2 = 4$

$\frac{9}{4} + y^2 = 4$

$4 - \frac{9}{4} = \frac{4 \cdot 4}{4 \cdot 1} - \frac{9}{4}$

$\frac{16}{4} - \frac{9}{4} = \frac{7}{4}$

$\sqrt{y^2} = \sqrt{\frac{7}{4}}$

$y = \pm \frac{\sqrt{7}}{2}$

$\boxed{\begin{array}{l} (3/2, \sqrt{7}/2) \\ (3/2, -\sqrt{7}/2) \end{array}}$

$x = -\frac{3}{2}$

$x^2 + y^2 = 4$

$\left(-\frac{3}{2}\right)^2 + y^2 = 4$

$\frac{9}{4} + y^2 = 4$

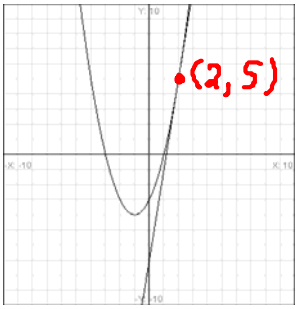
$\sqrt{y^2} = \sqrt{\frac{7}{4}}$

$y = \pm \frac{\sqrt{7}}{2}$

$\boxed{\begin{array}{l} (-3/2, \sqrt{7}/2) \\ (-3/2, -\sqrt{7}/2) \end{array}}$

4. $y = x^2 + 2x - 3$

$6x - y = 7$



$6x - y = 7$

$6x - 7 = y$

$y = \underline{6x - 7}$

$y = x^2 + 2x - 3$

$6x - 7 = x^2 + 2x - 3$

$x^2 - 4x + 4 = 0$

$(x-2)(x-2) = 0$

$x - 2 = 0$

$x = 2$

Substitute $x = 2$ into $y = 6x - 7$

$x = 2$

$y = 6x - 7$

$y = 6(2) - 7$

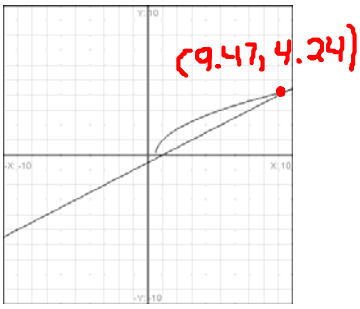
$y = 12 - 7$

$y = 5$

$\boxed{(2, 5)}$

5. $x-2y=1$

$y=\sqrt{2x-1}$



$y=\sqrt{2x-1}$ $\qquad$ $x-2y=1$

$$x-2\sqrt{2x-1}=1$$
$$-2\sqrt{2x-1}=1-x$$
$$\left(\sqrt{2x-1}\right)^2=\left(\frac{1-x}{-2}\right)^2$$

$$2x-1=\frac{(1-x)(1-x)}{4}$$
$$2x-1=\frac{1-x-x+x^2}{4}$$
$$2x-1=\frac{1-2x+x^2}{4}$$
$$4(2x-1)=1-2x+x^2$$
$$8x-4=1-2x+x^2$$
$$x^2-10x+5=0$$

$a=1 \quad b=-10 \quad c=5$

$$x=\frac{-b\pm\sqrt{b^2-4ac}}{2a}$$
$$x=\frac{-(-10)\pm\sqrt{(-10)^2-4(1)(5)}}{2(1)}$$
$$x=\frac{10\pm\sqrt{80}}{2}$$

$x=\frac{10+\sqrt{80}}{2}$ $\qquad$ $x=\frac{10-\sqrt{80}}{2}$

$x=9.47$ $\qquad$ $x=.53$

Substitute $x=9.47$ and $x=.53$ into $y=\sqrt{2x-1}$ and $x-2y=1$

$x=9.47$
$y=\sqrt{2x-1}$
$y=\sqrt{2(9.47)-1}$
$y=4.24$

$\boxed{(9.47, 4.24)}$

$x-2y=1$
$9.47-2(4.24)=1$
$1=1$ ✓

$x=.53$
$y=\sqrt{2x-1}$
$y=\sqrt{2(.53)-1}$
$y=.24$

$(.53, .24)$

$x-2y=1$
$.53-2(.24)=1$
$.06=1$ X