Multiplying Matrices

$$A_{2\times 3}$$

number of rows

number of columns

$$\overbrace{\begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \end{bmatrix}}^{\text{3 columns}} \left.\vphantom{\begin{bmatrix} a \\ a \end{bmatrix}}\right\} \text{2 rows}$$

$$A = \begin{bmatrix} 1 & -2 \\ -4 & 0 \\ 5 & -1 \end{bmatrix}$$

a_{12}

$A_{2\times 3} \bullet B_{3\times 4} = AB_{2\times 4}$

$2 \times \cancel{3} \quad \cancel{3} \times 4 = 2 \times 4$

$B_{3\times 4} \bullet A_{2\times 3} =$ undefined

$3 \times \underline{4} \quad \underline{2} \times 3 =$ undefined

must be the same

Directions: Find the dimensions of matrix *AB*.

1. $A_{3\times 1} \bullet B_{1\times 2}$

$3 \times \cancel{1} \quad \cancel{1} \times 2 = \boxed{3 \times 2}$

2. $A_{6\times 4} \bullet B_{4\times 2}$

$6 \times \cancel{4} \quad \cancel{4} \times 2 = \boxed{6 \times 2}$

3. $A_{1\times 4} \bullet B_{3\times 1}$

$1\times \underline{4} \quad 3\times \underline{1} = \text{undefined}$

Directions: Find each product, if possible.

4. $\begin{bmatrix} -2 & 0 & 9 \end{bmatrix} \bullet \begin{bmatrix} -12 & -10 \\ 4 & -1 \\ 8 & 5 \end{bmatrix} = \begin{bmatrix} \underline{96} & \underline{65} \end{bmatrix} = \boxed{\begin{bmatrix} 96 & 65 \end{bmatrix}}$

$1\times \cancel{3} \quad \cancel{3}\times 2 = 1\times 2$

a_{11}: $(-2)(-12) + (0)(4) + (9)(8) = 24 + 0 + 72 = 96$

a_{12}: $(-2)(-10) + (0)(-1) + (9)(5) = 20 + 0 + 45 = 65$

5. $\begin{bmatrix} 1 & -2 \\ -4 & 0 \\ 5 & -1 \end{bmatrix} \bullet \begin{bmatrix} 11 & 6 \\ -5 & 1 \end{bmatrix} = \begin{bmatrix} \underline{21} & \underline{4} \\ \underline{-44} & \underline{-24} \\ \underline{60} & \underline{29} \end{bmatrix} = \boxed{\begin{bmatrix} 21 & 4 \\ -44 & -24 \\ 60 & 29 \end{bmatrix}}$

$3\times \cancel{2} \quad \cancel{2}\times 2 = 3\times 2$

a_{11}: $(1)(11) + (-2)(-5) = 11 + 10 = 21$

a_{12}: $(1)(6) + (-2)(1) = 6 + -2 = 4$

a_{21}: $(-4)(11) + (0)(-5) = -44 + 0 = -44$

a_{22}: $(-4)(6) + (0)(1) = -24 + 0 = -24$

a_{31}: $(5)(11) + (-1)(-5) = 55 + 5 = 60$

a_{32}: $(5)(6) + (-1)(1) = 30 + -1 = 29$

6. $2\begin{bmatrix} -5 & 2 \\ 7 & -6 \end{bmatrix} \bullet \begin{bmatrix} 1 & -3 & 2 \\ 6 & 4 & -9 \end{bmatrix} = \begin{bmatrix} -10 & 4 \\ 14 & -12 \end{bmatrix} \bullet \begin{bmatrix} 1 & -3 & 2 \\ 6 & 4 & -9 \end{bmatrix}$

$2 \times \cancel{2} \quad \cancel{2} \times 3 = 2 \times 3$

$$\begin{bmatrix} \underline{14} & \underline{46} & \underline{-56} \\ \underline{-58} & \underline{-90} & \underline{136} \end{bmatrix} = \boxed{\begin{bmatrix} 14 & 46 & -56 \\ -58 & -90 & 136 \end{bmatrix}}$$

a_{11}: $(-10)(1) + (4)(6) = -10 + 24 = 14$

a_{12}: $(-10)(-3) + (4)(4) = 30 + 16 = 46$

a_{13}: $(-10)(2) + (4)(-9) = -20 + -36 = -56$

a_{21}: $(14)(1) + (-12)(6) = 14 + -72 = -58$

a_{22}: $(14)(-3) + (-12)(4) = -42 + -48 = -90$

a_{23}: $(14)(2) + (-12)(-9) = 28 + 108 = 136$

7. $\begin{bmatrix} 11 & 7 \\ 3 & 10 \\ 5 & 1 \end{bmatrix} \bullet \begin{bmatrix} 1 & -2 \\ -4 & 0 \\ 5 & -1 \end{bmatrix}$

$3 \times \underline{2} \quad \underline{3} \times 2 = \boxed{\text{undefined}}$

Directions: Use matrices A, B and C to simplify the expression.

$$A = \begin{bmatrix} 2 & 1 \\ 6 & -5 \end{bmatrix} \quad B = \begin{bmatrix} -30 \\ 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -5 \end{bmatrix}$$

8. $BC + A$

$$\underbrace{\begin{bmatrix} -30 \\ 1 \end{bmatrix} \cdot \begin{bmatrix} 1 & -5 \end{bmatrix}} + \begin{bmatrix} 2 & 1 \\ 6 & -5 \end{bmatrix}$$

$2 \times \cancel{1} \quad \cancel{1} \times 2 = 2 \times 2$

$$\begin{bmatrix} \underline{-30} & \underline{150} \\ \underline{1} & \underline{-5} \end{bmatrix} = \begin{bmatrix} -30 & 150 \\ 1 & -5 \end{bmatrix} + \begin{bmatrix} 2 & 1 \\ 6 & -5 \end{bmatrix}$$

a_{11}: $(-30)(1) = -30$

a_{12}: $(-30)(-5) = 150$

a_{21}: $(1)(1) = 1$

a_{22}: $(1)(-5) = -5$

$$\boxed{= \begin{bmatrix} -28 & 151 \\ 7 & -10 \end{bmatrix}}$$

Directions: Solve for x and y.

9. $\underbrace{\begin{bmatrix} 4 & 1 \\ -3 & -2 \end{bmatrix}}\bullet\begin{bmatrix} x \\ y \end{bmatrix}=\begin{bmatrix} 6 \\ 8 \end{bmatrix}$

$2\times\cancel{2}\quad \cancel{2}\times 1 = 2\times 1$

$$\begin{bmatrix} 4x+y \\ -3x-2y \end{bmatrix} = \begin{bmatrix} 4x+y \\ -3x-2y \end{bmatrix} \qquad \begin{bmatrix} 4x+y \\ -3x-2y \end{bmatrix} = \begin{bmatrix} 6 \\ 8 \end{bmatrix}$$

a_{11}: $(4)(x)+(1)(y) = 4x+y$

a_{21}: $(-3)(x)+(-2)(y) = -3x-2y$

$4x+y=6$

$-3x-2y=8$

$2(4x+y=6) \rightarrow$

$-3x-2y=8$

$$\begin{aligned} 8x+2y &= 12 \\ -3x-2y &= 8 \\ \hline 5x &= 20 \\ \boxed{x=4} \end{aligned}$$

$$\begin{aligned} 4x+y &= 6 \\ 4(4)+y &= 6 \\ 16+y &= 6 \\ \boxed{y=-10} \end{aligned}$$