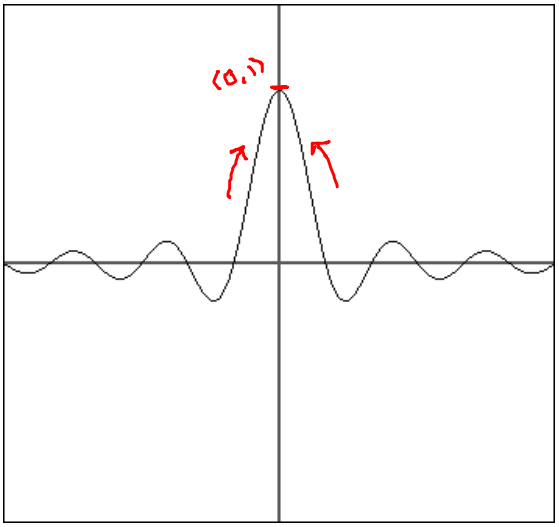
Limits of Trigonometric Functions

Step 1: Substitute the value into the limit.
Step 2: If the denominator equals zero, try one of the following techniques:

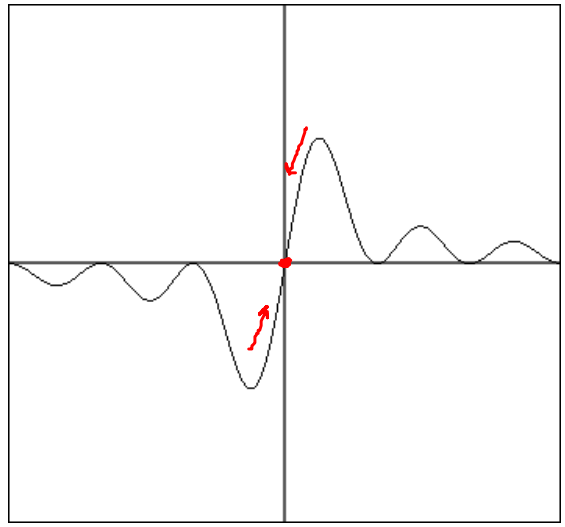
a) Apply a trigonometric identity.
b) Apply a special limit.

<u>Special Limits</u>

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$



$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$



Directions: Evaluate each limit.

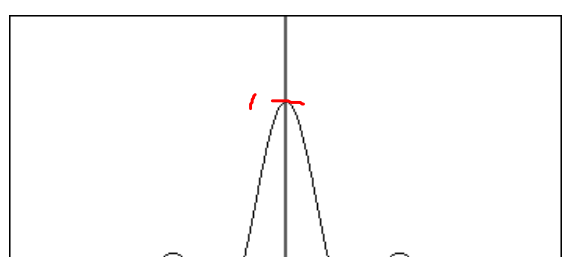
1) $\lim_{\theta\to 0}\sin\theta =$ $\sin 0 = \boxed{0}$

2) $\lim_{\theta\to 0}\cos\theta =$ $\cos 0 = \boxed{1}$

3) $\lim_{\theta\to 0}\tan\theta =$ $\tan 0 = \frac{\sin 0}{\cos 0} = \frac{0}{1} = \boxed{0}$

4) $\lim_{\theta\to 0}\frac{\sin\theta}{\theta} =$ $\boxed{1}$

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$



5) $\lim_{x\to\infty}\frac{\sin x}{x} =$ $\boxed{0}$

6) $\lim_{x\to 0}\frac{\sin 3x}{x} = \frac{\sin u}{\frac{u}{3}} = \sin u \div \frac{u}{3} = \sin u \cdot \frac{3}{u}$

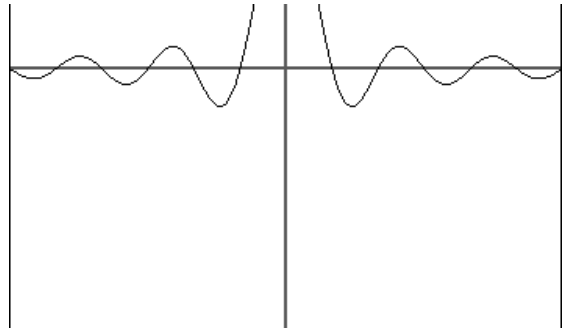
$u = 3x$

$x = \frac{u}{3}$

$\lim_{x\to 0} \sin u \cdot \frac{3}{u} = \frac{\sin u}{u} \cdot 3$

$= 1$

$= 1 \cdot 3 = \boxed{3}$



7) $\lim_{x\to 0}\frac{\tan x}{x} = \frac{\frac{\sin x}{\cos x}}{\frac{x}{1}} = \frac{\sin x}{\cos x} \div \frac{x}{1} = \frac{\sin x}{\cos x} \cdot \frac{1}{x} = \frac{\sin x}{x} \cdot \frac{1}{\cos x}$

$\lim_{x\to 0} \frac{\sin x}{x} \cdot \frac{1}{\cos x} = 1 \cdot \frac{1}{\cos 0} = 1 \cdot \frac{1}{1} = 1 \cdot 1 = \boxed{1}$

$= 1$

8) $\lim_{x\to 0}\frac{\sin^2 x}{x^2} = \frac{\sin x \cdot \sin x}{x \cdot x} = \frac{\sin x}{x} \cdot \frac{\sin x}{x} = 1 \cdot 1 = \boxed{1}$

9) $\lim_{x\to 0}\frac{\tan x}{x^2+1} = \frac{\tan 0}{0^2+1} = \frac{0}{1} = \boxed{0}$

10) $\lim_{x\to \pi} x \bullet \sec x = x \cdot \frac{1}{\cos x} = \frac{x}{\cos x}$

$\lim_{x\to \pi} \frac{x}{\cos x} = \frac{\pi}{\cos \pi} = \frac{\pi}{-1} = \boxed{-\pi}$

11) $\lim_{x\to 0}\frac{3(1-\cos x)}{x} = 3 \cdot 0 = \boxed{0}$

$= 0$

12) $\lim_{x\to \frac{\pi}{4}}\frac{1-\tan x}{\sin x - \cos x} = \frac{\frac{1 \cdot \cos x}{1 \cdot \cos x} - \frac{\sin x}{\cos x}}{}$

LCD $= \cos x$

$\frac{\cos x}{\cos x} - \frac{\sin x}{\cos x}$

$$\sin x - \cos x \qquad \frac{\sin x - \cos x}{1}$$

$$= \frac{\cos x - \sin x}{\cos x} \div \frac{\sin x - \cos x}{1} = \frac{\overset{-1}{\cancel{\cos x - \sin x}}}{\cos x} \cdot \frac{1}{\cancel{\sin x - \cos x}}$$

$$\lim_{x \to \pi/4} \frac{-1}{\cos x} = \frac{-1}{\cos 45^\circ} = \frac{-1}{\frac{1}{\sqrt{2}}} = -1 \div \frac{1}{\sqrt{2}} = -1 \times \frac{\sqrt{2}}{1} = \boxed{-\sqrt{2}}$$

$$\frac{\pi}{4} \cdot \frac{180}{\pi} = 45^\circ$$