

Definition of the Derivative

The derivative of a function denoted $f'(x)$ is equal to:

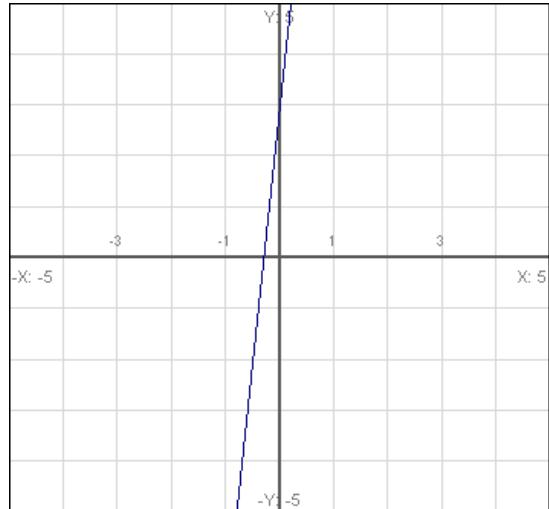
$$\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} \text{ or } \lim_{\Delta x \rightarrow 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

Directions: a) Use the Definition of the Derivative to find $f'(x)$.
 b) Using the result in part a, find $f'(-1)$, $f'(-\frac{1}{3})$, $f'(0)$ and $f'(1)$.

1. $f(x) = 10x + 3$

a) $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{10(x+h) + 3 - (10x+3)}{h} = \lim_{h \rightarrow 0} \frac{10h}{h} = 10$

b) $f'(-1) = 10$
 $f'(-\frac{1}{3}) = 10$
 $f'(0) = 10$
 $f'(1) = 10$



2. $f(x) = 3x^2 + 2x + 1$

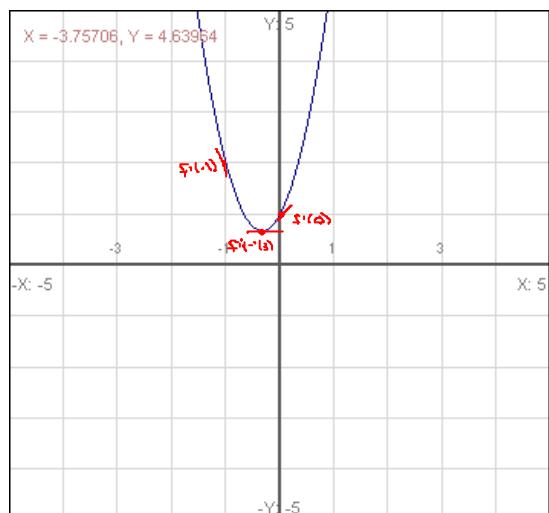
a) $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \lim_{h \rightarrow 0} \frac{3(x+h)^2 + 2(x+h) + 1 - (3x^2 + 2x + 1)}{h}$

$\lim_{h \rightarrow 0} \frac{3(x^2 + 2xh + h^2) + 2(x+h) + 1 - (3x^2 + 2x + 1)}{h}$

$\lim_{h \rightarrow 0} \frac{3x^2 + 6xh + 3h^2 + 2x + 2h + 1 - 3x^2 - 2x - 1}{h}$

$\lim_{h \rightarrow 0} \frac{h(6x + 3h + 2)}{h} = 6x + 3(0) + 2 = 6x + 2$

b) $f'(-1) = 6(-1) + 2 = -4$
 $f'(-\frac{1}{3}) = 6(-\frac{1}{3}) + 2 = 0$
 $f'(0) = 6(0) + 2 = 2$
 $f'(1) = 6(1) + 2 = 8$



$$3. f(x) = \sqrt{x}$$

a) $\lim_{h \rightarrow 0} \frac{f(x+h) - f(x)}{h} = \frac{\sqrt{x+h} - \sqrt{x}}{h} \cdot \frac{\sqrt{x+h} + \sqrt{x}}{\sqrt{x+h} + \sqrt{x}}$

$$\lim_{h \rightarrow 0} \frac{(\sqrt{x+h} - \sqrt{x})(\sqrt{x+h} + \sqrt{x})}{h(\sqrt{x+h} + \sqrt{x})} =$$

$$\lim_{h \rightarrow 0} \frac{1}{\sqrt{x+h} + \sqrt{x}} = \boxed{\frac{1}{2\sqrt{x}}}$$

b) $f'(-1) = \underline{\quad} =$ Does not exist

21-1 (can't have a negative number under the square root)

$$f'(-1) = \underline{\underline{1}} = \text{Does not exist}$$

$2\sqrt{-13}$ (can't have a negative number under the square root)

$$f'(0) = \underline{1} = \text{Does not exist}$$

250 (can't have a zero in the denominator)

$$f'(1) = \frac{1}{2\sqrt{1}} = \boxed{\frac{1}{2}}$$

