Product Rule and Quotient Rule

Product Rule

If $P(x) = \underset{1st}{f(x)} \bullet \underset{2nd}{g(x)}$ then $P'(x) = \underset{1st}{f'(x)} \bullet \underset{2nd}{g(x)} + \underset{1st}{f(x)} \bullet \underset{2nd}{g'(x)}$

Directions: Find the derivative of each.

1. $f(x) = \underset{1st}{\left(x^2+1\right)}\underset{2nd}{\left(2x+3\right)}$

By Product Rule

$$f'(x) = \underset{f'(x)}{(2x)}\underset{g(x)}{(2x+3)} + \underset{f(x)}{(x^2+1)}\underset{g'(x)}{(2)}$$
$$= 4x^2+6x+2x^2+2$$
$$= \boxed{6x^2+6x+2}$$

By FOIL Method

$$f(x) = (x^2+1)(2x+3)$$
$$f(x) = 2x^3+3x^2+2x+3$$
$$f'(x) = \boxed{6x^2+6x+2}$$

2. Let $f(x) = \underset{1st}{x^4} \bullet \underset{2nd}{g(x)}$. If $g(2) = 3$ and $g'(2) = 5$, then find $f'(2)$.

$$f'(x) = (4x^3)(g(x)) + (x^4)(g'(x))$$

$$f'(2) = (4 \cdot 2^3)(g(2)) + (2^4)(g'(2))$$
$$= (32)(3) + (16)(5)$$
$$= 96 + 80$$
$$= \boxed{176}$$

Quotient Rule

If $Q(x) = \dfrac{\overset{\text{Num}}{f(x)}}{\underset{\text{DEN}}{g(x)}}$ then $Q'(x) = \dfrac{\overset{N}{f'(x)} \bullet \overset{D}{g(x)} - \overset{N}{f(x)} \bullet \overset{D}{g'(x)}}{\underset{D}{\left[g(x)\right]^2}}$

Directions: Find the derivative of each.

1. $f(x) = \dfrac{6x+1}{3x-2}$

$$f'(x) = \frac{\overset{N}{(6)}\overset{D}{(3x-2)} - \overset{N}{(6x+1)}\overset{D}{(3)}}{} = \cancel{18x} - 12 - \cancel{18x} - 3 = \boxed{\frac{-15}{\ }}$$

$(3x-2)^2$

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$\overline{\qquad\qquad}$ $(3x-2)^2$ $\boxed{(3x-2)^2}$

2. $f(x)=\dfrac{3x-5}{x^2+7}$

$$f'(x)=\frac{(3)(x^2+7)-(3x-5)(2x)}{(x^2+7)^2}=\frac{3x^2+21-6x^2+10x}{(x^2+7)^2}=\boxed{\frac{-3x^2+10x+21}{(x^2+7)^2}}$$