Derivatives of Trigonometric Functions

$\frac{d}{dx}\sin x=\cos x$ $\qquad \frac{d}{dx}\sec x=\sec x\tan x$

$\frac{d}{dx}\cos x=-\sin x$ $\qquad \frac{d}{dx}\csc x=-\csc x\cot x$

$\frac{d}{dx}\tan x=\sec^2 x$ $\qquad \frac{d}{dx}\cot x=-\csc^2 x$

Product Rule

$P'(x)=\underset{1^{st}}{f'(x)}\bullet \underset{2^{nd}}{g(x)}+\underset{1^{st}}{f(x)}\bullet \underset{2^{nd}}{g'(x)}$

Quotient Rule

$Q'(x)=\frac{\overset{N}{f'(x)}\bullet \overset{D}{g(x)}-\overset{N}{f(x)}\bullet \overset{D}{g'(x)}}{\underset{D}{[g(x)]^2}}$

Directions: Find the derivative of each.

1) $y=x^2-\sin x$

$\boxed{y'=2x-\cos x}$

2) $y=\overset{1^{st}}{x^2}\bullet\overset{2^{nd}}{\sin x}$

$y'=(2x)(\sin x)+(x^2)(\cos x)$

$\boxed{y'=2x\sin x+x^2\cos x}$

3) $y=\frac{\overset{N}{\sin x}}{\underset{D}{x}}$

$y'=\frac{(\cos x)(x)-(\sin x)(1)}{x^2}$

$\boxed{y'=\frac{x\cos x-\sin x}{x^2}}$

4) $y=\sin x\bullet\cos x$

$y'=(\cos x)(\cos x)+(\sin x)(-\sin x)$

$y'=\cos^2 x-\sin^2 x$

$\boxed{y'=\cos 2x}$

$\cos 2x=\cos^2 x-\sin^2 x$

5) $y=\frac{\cos x}{1-\sin x}$

$y'=\frac{(-\sin x)(1-\sin x)-(\cos x)(-\cos x)}{(1-\sin x)^2}$

$y'=\frac{-\sin x+\overbrace{\sin^2 x+\cos^2 x}^{1}}{(1-\sin x)^2}$

$y'=\frac{\overset{1}{\cancel{1-\sin x}}}{(1-\sin x)^{\cancel{2}\,1}}$

$\boxed{y'=\frac{1}{1-\sin x}}$

6) $y=\frac{2}{\sin x}$

$y=\quad cx$

$y'=2(-\csc x\cot x)$

$\boxed{y'=-2\csc x\cot x}$

Directions: Find y''.

7) $y=\sec x$

$y'=\sec x\cdot\tan x$

$y''=(\sec x\tan x)\tan x+\sec x(\sec^2 x)$

$y''=\sec x\tan^2 x+\sec^3 x$

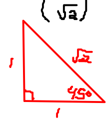
$\boxed{y''=\sec x(\tan^2 x+\sec^2 x)}$

8) Find the lines that are tangent and normal to the curve

$y=\tan x$ at $\left(\frac{\pi}{4},1\right)$ in slope-intercept form.

$y'=\sec^2 x\Big|_{(\pi/4,1)}=\frac{1}{\cos^2 x}=\frac{1}{\cos^2(\frac{\pi}{4})}=\frac{1}{\cos^2(45^\circ)}=\frac{1}{\left(\frac{1}{\sqrt{2}}\right)^2}=\frac{1}{\frac{1}{2}}=2$

$\frac{\pi}{4}=\frac{180}{4}=45^\circ$



Equation of Tangent Line:

$x_1 \quad y_1$

Equation of Normal Line:

$x_1 \quad y_1$

LINE.   LINE.

$m_T = 2 \quad (\overset{x_1}{\pi/4}, \overset{y_1}{1})$

$y - y_1 = m(x - x_1)$

$y - 1 = 2(x - \frac{\pi}{4})$

$y - 1 = 2x - \frac{\pi}{2}$

$+1 \qquad +1$

$\boxed{y = 2x - \pi/2 + 1}$

$m_N = -\frac{1}{2} \quad (\overset{x_1}{\pi/4}, \overset{y_1}{1})$

$y - y_1 = m(x - x_1)$

$y - 1 = -\frac{1}{2}(x - \pi/4)$

$y - 1 = -\frac{1}{2}x + \pi/8$

$+1 \qquad +1$

$\boxed{y = -\frac{1}{2}x + \pi/8 + 1}$

Directions: Find the derivative of each using the chain rule.

9) $y = \sin(2x)$

$y' = (\cos 2x)(2)$

$\boxed{y' = 2\cos 2x}$

10) $y = \cos(3x^2)$

$y' = [-\sin(3x^2)](6x)$

$y' = \boxed{-6x\sin(3x^2)}$

11) $y = \cos^2(3x) = [\cos(3x)]^2$

$y' = (2\cos 3x)(-\sin(3x))(3)$

$\boxed{y' = -6\cos 3x \sin 3x}$

12) $y = \sin^3(4t)$

$y = [\sin(4t)]^3$

$y' = 3[\sin(4t)]^2[\cos(4t)](4)$

$\boxed{y' = 12(\sin(4t))^2\cos(4t)}$

13) Find the derivative of the trigonometric equation using implicit differentiation.

$\underbrace{x \cdot \sin y} = \underbrace{y \cdot \cos x}$

$(1)\sin y + x(\cos y)y' = (1)y'(\cos x) + y(-\sin x)$

$\sin y + y'x\cos y = y'\cos x - y\sin x$

$\quad -y'\cos x \qquad -\cancel{y'\cos x}$

$\cancel{\sin y} + y'x\cos y - y'\cos x = -y\sin x$

$-\cancel{\sin y} \qquad -\sin y$

$y'x\cos y - y'\cos x = -y\sin x - \sin y$

$\frac{y'(\cancel{x\cos y - \cos x})}{\cancel{x\cos y - \cos x}} = \frac{-y\sin x - \sin y}{x\cos y - \cos x}$

$\boxed{y' = \frac{-y\sin x - \sin y}{x\cos y - \cos x}}$