Higher Order Derivatives

If $y = f(x)$ then the derivative of y with respect to x is denoted by:

$$f'(x) \qquad \frac{dy}{dx} \qquad y'$$

The second derivative is the derivative of the first derivative and is denoted by:

$$f''(x) \qquad \frac{d^2y}{dx^2} \qquad y''$$

The third derivative is the derivative of the second derivative and is denoted by:

$$f'''(x) \qquad \frac{d^3y}{dx^3} \qquad y'''$$

The nth derivative ($n > 3$) is denoted by:

$$f^{(n)}(x) \qquad \frac{d^{(n)}y}{dx^{(n)}} \qquad y^{(n)}$$

1. Find the first four derivatives.
   $f(x) = 5x^4 - 3x^3 + 7x^2 - 5x + 3$

$f'(x) = 20x^3 - 9x^2 + 14x - 5$
$f''(x) = 60x^2 - 18x + 14$
$f'''(x) = 120x - 18$
$f^{(4)}(x) = 120$

2. Find $f'''(4)$ if $f(x) = \sqrt{x}. = x^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}x^{\frac{1}{2}-1} = \frac{1}{2}x^{-\frac{1}{2}}$
$f''(x) = -\frac{1}{4}x^{-\frac{1}{2}-1} = -\frac{1}{4}x^{-\frac{3}{2}}$
$f'''(x) = \frac{3}{8}x^{-3/2-1} = \frac{3}{8}x^{-5/2} = \frac{3}{8}\cdot\frac{1}{x^{5/2}} = \frac{3}{8(\sqrt{x})^5}$

$f'''(4) = \frac{3}{8(\sqrt{4})^5} = \frac{3}{8(32)} = \boxed{\frac{3}{256}}$

3. Find $f''(2)$ if $f(x) = x^2 + \frac{1}{x}. = x^2 + x^{-1}$

$f'(x) = 2x - x^{-2}$
$f''(x) = 2 + 2x^{-3} = 2 + \frac{2}{x^3}$

$$f''(x) = 2 + 2x^{-3} = 2 + \frac{2}{x^3}$$

$$f''(2) = 2 + \frac{2}{2^3} = 2 + \frac{2}{8} = 2 + \frac{1}{4} = \boxed{2\tfrac{1}{4} \text{ OR } \tfrac{9}{4}}$$

4. Find $f''(x)$ if $f(x) = \dfrac{2x-1}{3x+2}$.

$$f'(x) = \frac{(2)(3x+2) - (2x-1)(3)}{(3x+2)^2} = \frac{\cancel{6x} + 4 - \cancel{6x} + 3}{(3x+2)^2} = \frac{7}{(3x+2)^2} = 7(3x+2)^{-2}$$

$$f''(x) = -14\underbrace{(3x+2)}^{-3}(3) = \boxed{\frac{-42}{(3x+2)^3}}$$

5. Find y'' if $x^2 + y^2 = 16$.

$$2x + 2yy' = 0$$
$$-2x \qquad -2x$$
$$\frac{2yy'}{2y} = \frac{-2x}{2y}$$
$$y' = \frac{-x}{y}$$

$$y' = \frac{-x}{y}$$

$$y'' = \frac{(-1)(y) - (-x)(1)y'}{y^2}$$

$$y'' = \frac{-y + x\left(\frac{-x}{y}\right)}{y^2}$$

$$y'' = \frac{\frac{-y \cdot y}{1 \cdot y} - \frac{x^2}{y}}{y^2}$$

LCD = y

$$y'' = \frac{\frac{-y^2 - x^2}{y}}{\frac{y^2}{1}} \qquad \frac{-y^2 - x^2}{y} \cdot \frac{1}{y^2} = \frac{-y^2 - x^2}{y^3}$$

$$\boxed{y'' = \frac{-y^2 - x^2}{y^3}}$$

6. Find y'' if $x^2 + xy = 5$.

$$2x + (1y + x(1)y') = 0$$
$$2x + y + xy' = 0$$
$$-2x - y \qquad -2x - y$$
$$xy' = -2x - y$$

$$y' = \frac{-2x - y}{x}$$

$$y'' = \frac{(-2 - 1y')(x) - (-2x - y)(1)}{x^2}$$

$$\frac{xy'}{x} = \frac{-2x-y}{x}$$

$$y' = \frac{-2x-y}{x}$$

$$x^2$$

$$y'' = \frac{-\cancel{2x} - xy' \cancel{+2x} + y}{x^2}$$

$$y'' = \frac{-xy' + y}{x^2}$$

$$y'' = \frac{-\cancel{x}\left(\frac{-2x-y}{\cancel{x}}\right) + y}{x^2}$$

$$\frac{2x+y+y}{x^2}$$

$$\boxed{y'' = \frac{2x+2y}{x^2}}$$