

## L'Hopital's Rule

Use L'Hopital's Rule when the following conditions are met:

1)  $\lim_{x \rightarrow c} \frac{f(x)}{g(x)} = \frac{0}{0}$  or  $\frac{\infty}{\infty}$

2)  $f'(c)$  and  $g'(c)$  exist

3)  $g'(c) \neq 0$

Step 1: Substitute the value into the limit.

Step 2: Apply L'Hopital's Rule until the denominator does not equal zero.

Directions: Find the derivative of each.

1.  $\lim_{x \rightarrow 0} \frac{\sin x}{x} = \frac{\sin 0}{0} = \frac{0}{0}$

$\lim_{x \rightarrow 0} \frac{\cos x}{1} = \frac{\cos 0}{1} = \frac{1}{1} = \boxed{1}$

2.  $\lim_{x \rightarrow 0} \frac{2x - \sin x}{x} = \frac{2(0) - \sin 0}{0} = \frac{0-0}{0} = 0$

$\lim_{x \rightarrow 0} \frac{2 - \cos x}{1} = \frac{2 - \cos 0}{1} = \frac{2-1}{1} = \frac{1}{1} = \boxed{1}$

3.  $\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2}{2x} = \frac{\sqrt{4+0} - 2}{2(0)} = \frac{\sqrt{4} - 2}{0} = \frac{2-2}{0} = \frac{0}{0}$

$\lim_{x \rightarrow 0} \frac{\frac{1}{2}(\sqrt{4+x})^{-\frac{1}{2}}(1) - 0}{2} = \frac{\frac{1}{2\sqrt{4+x}}}{2} = \frac{1}{4\sqrt{4+x}} = \frac{1}{4\sqrt{4+0}} = \boxed{\frac{1}{8}}$

4.  $\lim_{x \rightarrow 0} \frac{\sqrt{4+x} - 2 - \frac{x}{4}}{2x^2} = \frac{\sqrt{4+x} - 2 - \frac{0}{4}}{2(0)^2} = \frac{\sqrt{4} - 2 - 0}{0} = \frac{2-2-0}{0} = \frac{0}{0}$

$\lim_{x \rightarrow 0} \frac{\frac{1}{2}(\sqrt{4+x})^{-\frac{1}{2}}(1) - 0 - \frac{1}{4}}{4x} = \frac{\frac{1}{2\sqrt{4+x}} - \frac{1}{4}}{4x} = \frac{\frac{1}{2\sqrt{4+0}} - \frac{1}{4}}{4(0)} = \frac{\frac{1}{4} - \frac{1}{4}}{0} = \frac{0}{0}$

...  
4x      - (v)

Apply L'Hopital's Rule again to  $\lim_{x \rightarrow 0} \frac{\frac{1}{2}(4+x)^{-\frac{1}{2}} - \frac{1}{4}}{4x}$

$$\lim_{x \rightarrow 0} \frac{-\frac{1}{4}(4+x)^{-\frac{3}{2}}(1) - 0}{4} = \frac{-\frac{1}{4}}{4(4+x)^{\frac{5}{2}}} = \frac{-\frac{1}{4}}{4(4+0)^{\frac{5}{2}}} = \frac{-\frac{1}{4}}{16(8)} = \boxed{-\frac{1}{128}}$$

5.  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{\sec x}{1 + \sec x}$  Rewrite  $\sec x = \frac{1}{\cos x}$

$$\lim_{x \rightarrow \frac{\pi}{2}} \frac{\frac{1}{\cos x}}{\frac{1}{\cos x} + \frac{1}{\cos x}} = \frac{\frac{1}{\cos x}}{\frac{2}{\cos x} + \frac{1}{\cos x}} = \frac{\frac{1}{\cos x}}{\frac{2 + 1}{\cos x}} = \frac{\frac{1}{\cos x}}{\frac{3}{\cos x}} = \frac{1}{3} = \boxed{1}$$

$\text{LCD} = \cos x$

6.  $\lim_{x \rightarrow 0^+} x \cdot \cot x$  Rewrite  $\cot x = \frac{1}{\tan x}$   $\lim_{x \rightarrow 0^+} \frac{x}{\tan x} = \frac{0}{\tan 0} = \frac{0}{0}$

$$\lim_{x \rightarrow 0^+} \frac{1}{\sec^2 x} = \cos^2 x = \cos^2 0 = (1)^2 = \boxed{1}$$

7.  $\lim_{x \rightarrow 0} \frac{\sin 8x}{x} = \frac{\sin 8(0)}{0} = \frac{\sin 0}{0} = \frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{(\cos 8x)(8)}{1} = 8 \cos 8x = 8 \cos(8 \cdot 0) = 8 \cos 0 = 8 \cdot 1 = \boxed{8}$$

8.  $\lim_{x \rightarrow 0} \frac{\tan x}{x} = \frac{\tan 0}{0} = \frac{0}{0}$

$$\lim_{x \rightarrow 0} \frac{\sec^2 x}{1} = \frac{1}{\cos^2 x} = \frac{1}{\cos^2 0} = \frac{1}{1^2} = \boxed{1}$$

9.  $\lim_{x \rightarrow \frac{\pi}{2}} \frac{x - \frac{\pi}{2}}{\cos x} = \frac{\frac{\pi}{2} - \frac{\pi}{2}}{\cos \frac{\pi}{2}} = \frac{0}{0}$

$$\lim_{x \rightarrow \pi/2} \frac{1 - 0}{-\sin x} = \frac{1}{-\sin x} = \frac{1}{-\sin \pi/2} = \frac{1}{-1} = \boxed{-1}$$