Velocity and Acceleration

Velocity - The first derivative
How fast the position is changing
The derivative of the position function $s(t)$

Speed - The absolute value of the first derivative

Acceleration - The second derivative
How fast the velocity is changing
How quickly the body loses or picks up speed
The derivative of the velocity function $v(t)$

1. A particle moves along the x-axis so that its position at time t is given by $s(t) = 3t^3 - 40.5t^2 + 162t$ for $0 \le t \le 8$.

a) Calculate the initial velocity.
b) When is the particle moving to the right?
c) When is the particle moving to the left?
d) When is the object decelerating?
e) When is the velocity not changing?
f) When is the object accelerating?

$s(t) = 3t^3 - 40.5t^2 + 162t$
$v(t) = 9t^2 - 81t + 162$
$a(t) = 18t - 81$

a) $v(0) = 9(0)^2 - 81(0) + 162$
$= 0 - 0 + 162$
$= \boxed{162}$

b) $v(t) > 0$
$\frac{9t^2}{9} - \frac{81t}{9} + \frac{162}{9} > \frac{0}{9}$
$t^2 - 9t + 18 = 0$
$(t-6)(t-3) = 0$
$t = 6 \quad t = 3$

$\boxed{t < 3 \text{ or } t > 6}$

c) $v(t) < 0$
$\frac{9t^2}{9} - \frac{81t}{9} + \frac{162}{9} < \frac{0}{9}$
$t^2 - 9t + 18 = 0$
$(t-6)(t-3) = 0$
$t = 6 \quad t = 3$

$\boxed{3 < t < 6}$

d) $a(t) < 0$
$18t - 81 < 0$
$+81 \quad +81$
$\frac{18t}{18} < \frac{81}{18}$
$\boxed{t < 4.5}$

e) $a(t) = 0$
$18t - 81 = 0$
$\boxed{t = 4.5}$

f) $a(t) > 0$
$18t - 81 > 0$
$\boxed{t > 4.5}$

2. A particle moves along a straight line so that the position at time t is given by $s(t) = t^2 + 4t + 4$.
What is the acceleration of the particle at $t = 4$?

$s(t) = t^2 + 4t + 4$
$v(t) = 2t + 4$
$a(t) = 2$
$a(4) = \boxed{2}$

3. A particle moves along the x-axis so that at time t its position is given by $x(t) = t^3 - 6t^2 + 9t + 11$.

a) What is the velocity of the particle at $t = 0$?
b) During what time intervals is the particle moving to the left?
c) What is the total distance traveled by the particle from $t = 0$ to $t = 2$?

$s(t) = t^3 - 6t^2 + 9t + 11$
$v(t) = 3t^2 - 12t + 9$
$a(t) = 6t - 12$

a) $v(0) = 3(0)^2 - 12(0) + 9$
$= \boxed{9}$

b) $v(t) < 0$
$\frac{3t^2}{3} - \frac{12t}{3} + \frac{9}{3} < \frac{0}{3}$
$t^2 - 4t + 3 = 0$
$(t - 3)(t - 1) = 0$
$t = 3 \quad t = 1$

$\boxed{1 < t < 3}$

c) $s(2) = (2)^3 - 6(2)^2 + 9(2) + 11$
$= 8 - 6(4) + 18 + 11$
$= 8 - 24 + 18 + 11$
$= \boxed{13}$

4. A rock is launched in the air with a launch velocity of 160 feet/second. $v(0) = 160$
It reaches a height of $s = 160t - 16t^2$ feet after t seconds.

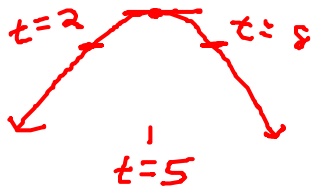
a) What is the maximum height of the rock?

b) How fast is the rock traveling when it is 256 feet above the ground on the way up and on the way down?

$s(t) = 160t - 16t^2$

$v(t) = 160 - 32t$

$a(t) = -32$



a) $v(t) = 0$

$160 - 32t = 0$

$-160 \quad -160$

$\frac{-32t}{-32} = \frac{-160}{-32}$

$t = 5$

$s(5) = 160(5) - 16(5)^2$

$= 800 - 400$

$= \boxed{400 \text{ feet}}$

b) $s(t) = 256$

$160t - 16t^2 = 256$

$\frac{-16t^2}{-16} + \frac{160t}{-16} - \frac{256}{-16} = \frac{0}{-16}$

$t^2 - 10t + 16 = 0$

$(t-8)(t-2) = 0$

$t = 8 \quad t = 2$

$v(2) = 160 - 32(2) = 160 - 64 = 96$

$= \boxed{96 \text{ ft/sec}}$

$v(8) = 160 - 32(8) = 160 - 256 = -96$

$= |-96| = \boxed{96 \text{ ft/sec}}$