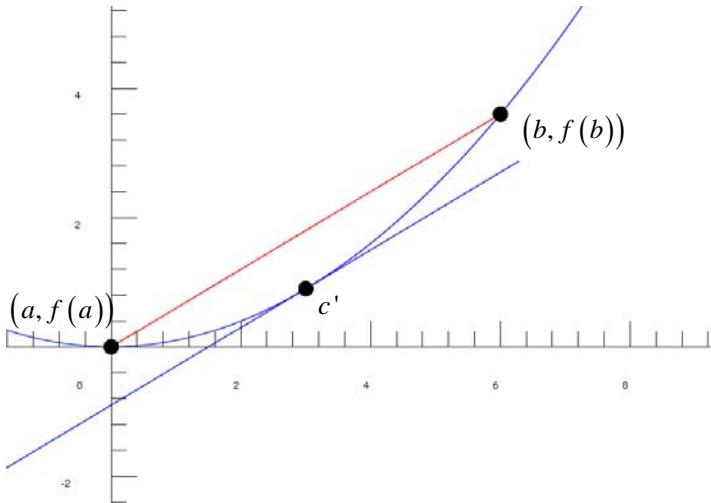


Mean Value Theorem

If a function is continuous on $[a, b]$ and differentiable on (a, b) then there exists a number c such that

$$f'(c) = \frac{f(b) - f(a)}{b - a}$$



- Find the value of c that satisfies the conclusion of the Mean Value Theorem for the given function on the given interval.

a) $f(x) = x^3 - x^2 - 2x$ on the interval $(-1, 1)$

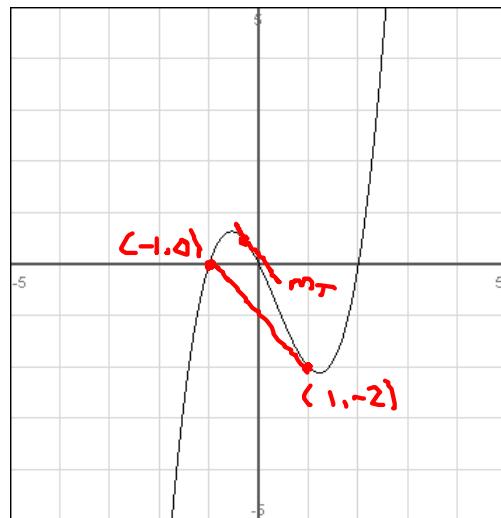
$$\begin{aligned} f(-1) &= (-1)^3 - (-1)^2 - 2(-1) \\ &= -1 - 1 + 2 \\ &= 0 \end{aligned}$$

$$\begin{aligned} f(1) &= 1^3 - (1)^2 - 2(1) \\ &= 1 - 1 - 2 \\ &= -2 \end{aligned}$$

$$(-1, 0) \quad (1, -2)$$

$$m = \frac{-2 - 0}{1 - (-1)} = \frac{-2}{2} = -1$$

$$f'(x) = 3x^2 - 2x - 2$$



$$3x^2 - 2x - 2 = -1$$

$$\begin{aligned} 3x^2 - 2x - 1 &= 0 \\ (3x + 1)(x - 1) &= 0 \end{aligned}$$

$$\begin{aligned} x &= -\frac{1}{3} \quad x = 1 \\ I &\subset \boxed{-\frac{1}{3}} \end{aligned}$$

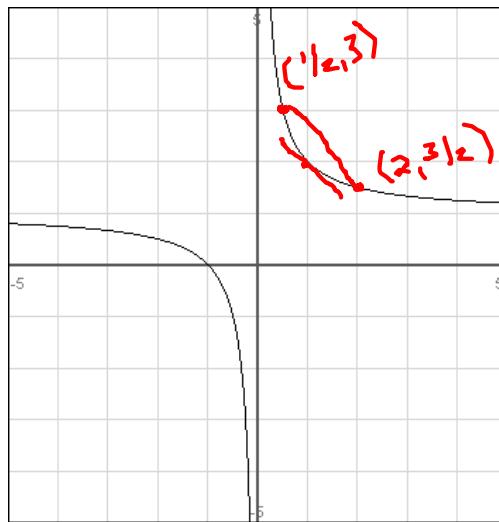
b) $f(x) = \frac{x+1}{x}$ on the interval $\left[\frac{1}{2}, 2\right]$

$$f\left(\frac{1}{2}\right) = \frac{\frac{1}{2} + 1}{\frac{1}{2}} = \frac{\frac{3}{2}}{\frac{1}{2}} = 3$$

$$f(2) = \frac{2+1}{2} = \frac{3}{2}$$

$(\frac{1}{2}, 3)$ $(2, \frac{3}{2})$

$$m = \frac{\frac{3}{2} - 3}{2 - \frac{1}{2}} = \frac{-\frac{3}{2}}{\frac{3}{2}} = -1$$



$$\begin{aligned} \frac{-1}{x^2} &= -\frac{1}{1} \\ -x^2 &= -1 \\ x^2 &= 1 \\ x &= \pm 1 \end{aligned}$$

$$c = 1$$

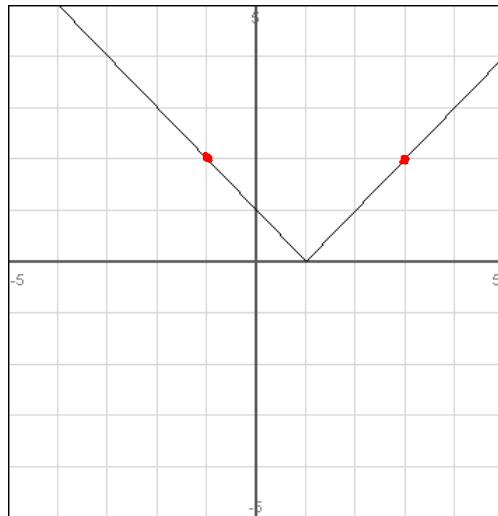
2. Determine if the Mean Value Theorem applies for the given function over the given interval.

a) $f(x) = |x-1|$ on $[-1, 3]$

$[-1, 3]$ continuous

$(-1, 3)$ not differentiable
at $x=1$

MVT does not apply

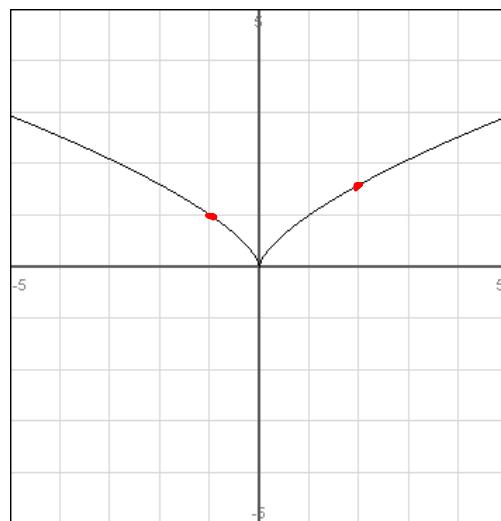


b) $f(x) = x^{\frac{2}{3}}$ on $[-1, 2]$

$[-1, 2]$ continuous

$(-1, 2)$ not differentiable
at $x=0$

MVT does NOT apply



c) $f(x) = \frac{1}{x-2}$ on $[3, 5]$

$[3, 5]$ continuous

$(3, 5)$ differentiable

MVT does apply

