

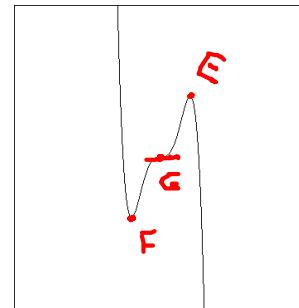
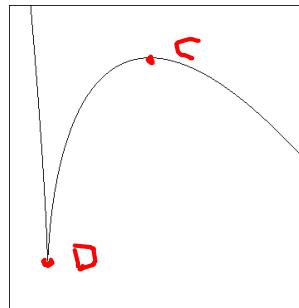
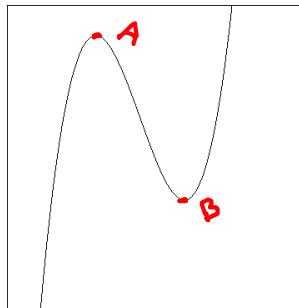
## Critical Points

Critical Point - A point  $(x_0, f(x_0))$  is called a critical point if:

- a)  $x_0$  is in the domain of  $f$ .
- b)  $f'(x_0) = 0$  or  $f'(x_0)$  does not exist.

### Types of Critical Points:

- a) Relative Extrema/Relative Maximum Point - A, C, E
- b) Relative Extrema/Relative Minimum Point - B, D, F
- c) Terrace Point - A critical point that is not a relative extrema - G



- For each function, find the critical points of  $f(x)$ .

a)  $f(x) = 2x^3 + 3x^2 - 12x$        $D: \mathbb{R}$

$$f'(x) = 6x^2 + 6x - 12$$

Critical numbers:  $f'(x) = 0$

$$\frac{6x^2}{6} + \frac{6x}{6} - \frac{12}{6} = 0$$

$$x^2 + x - 2 = 0$$

$$(x+2)(x-1) = 0$$

$$x+2=0 \quad x-1=0$$

$$x = -2 \quad x = 1$$

$$f(x) = 2x^3 + 3x^2 - 12x$$

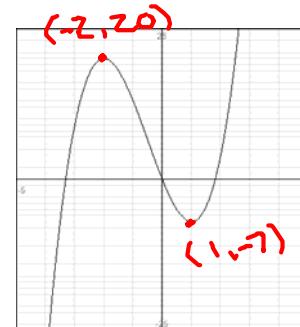
$$f(-2) = 2(-2)^3 + 3(-2)^2 - 12(-2)$$

$$= 2(-8) + 3(4) + 24$$

$$= -16 + 12 + 24$$

$$= 20$$

$$\boxed{(-2, 20)}$$



$$\begin{aligned}
 f(1) &= 2(1)^3 + 3(1)^2 - 12(1) \\
 &= 2 + 3 - 12 \\
 &= -7 \\
 \boxed{(1, -7)}
 \end{aligned}$$

$$b) f(x) = (x^2 - 4)^{\frac{2}{3}}$$

D:  $\mathbb{R}$

$$f'(x) = \frac{2}{3} (x^2 - 4)^{-\frac{1}{3}} (2x) = \frac{4x}{3\sqrt[3]{x^2 - 4}}$$

$$\underline{f'(x) = 0}$$

$$4x = 0$$

$$x = 0$$

$$\underline{f'(x) \text{ DNE}}$$

$$\frac{3}{3} \sqrt[3]{x^2 - 4} = 0$$

$$(\sqrt[3]{x^2 - 4})^3 = (0)^3$$

$$x^2 - 4 = 0$$

$$+4 +4$$

$$x^2 = 4$$

$$x = \pm 2$$

$$f(x) = (x^2 - 4)^{\frac{2}{3}}$$

$$f(0) = (0^2 - 4)^{\frac{2}{3}}$$

$$f(0) = (-4)^{\frac{2}{3}}$$

$$f(0) = \sqrt[3]{16} = 2\sqrt[3]{2}$$

$$\boxed{(0, 2\sqrt[3]{2})}$$

$$f(-2) = ((-2)^2 - 4)^{\frac{2}{3}}$$

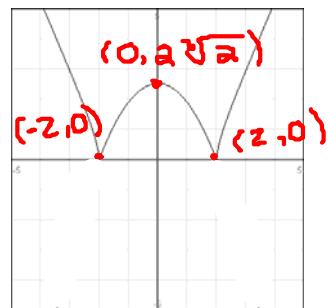
$$f(-2) = 0$$

$$\boxed{(-2, 0)}$$

$$f(2) = (2^2 - 4)^{\frac{2}{3}}$$

$$f(2) = 0$$

$$\boxed{(2, 0)}$$



$$c) f(x) = x + \frac{1}{x}$$

$$D: x \neq 0$$

$$f(x) = x + x^{-1}$$

$$f'(x) = 1 - 1x^{-2} = 1 - \frac{1}{x^2} = \frac{x^2 - 1}{x^2}$$

$$\underline{f'(x) = 0}$$

$$\frac{x^2 - 1}{x^2} = 0$$

$$+1 +1$$

$$x^2 = 1$$

$$x = \pm 1$$

$$f(-1) = -1 + \frac{1}{-1}$$

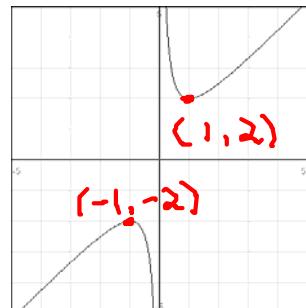
$$= -1 - 1$$

$$= -2$$

$$\boxed{(-1, -2)}$$

$$\underline{f'(x) \text{ DNE}}$$

$$\cancel{x = 0}$$



$$f(1) = 1 + \frac{1}{1}$$

$$= 1 + 1$$

$$= 2$$

$$\boxed{(1, 2)}$$

$$d) f(x) = 12x^{\frac{2}{3}} - 4x$$

D:  $\mathbb{R}$

$$f'(x) = 12 \cdot \frac{2}{3} x^{-\frac{1}{3}} - 4 = \frac{8}{\sqrt[3]{x}} - 4$$

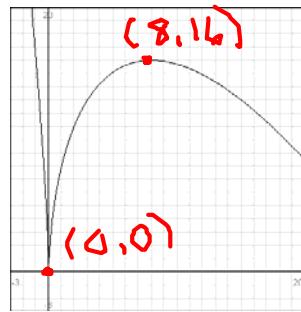
$$f'(x) = \frac{8 - 4\sqrt[3]{x}}{\sqrt[3]{x}}$$

$$\begin{aligned} f'(x) &= 0 \\ \frac{8 - 4\sqrt[3]{x}}{+ 4\sqrt[3]{x}} &= 0 \\ + 4\sqrt[3]{x} &+ 4\sqrt[3]{x} \end{aligned}$$

$$\begin{aligned} f'(x) &\text{ DNE} \\ \sqrt[3]{x} &= 0 \\ x &= 0 \end{aligned}$$

$$\begin{aligned} \frac{8}{4} &= \frac{4\sqrt[3]{x}}{4} \\ (2)^3 &= (\sqrt[3]{x})^3 \\ 8 &= x \\ x &= 8 \end{aligned}$$

$$\begin{aligned} f(x) &= 12x^{\frac{2}{3}} - 4x \\ f(8) &= 12(8)^{\frac{2}{3}} - 4(8) \\ f(8) &= 12(4) - 32 \\ f(8) &= 48 - 32 \\ f(8) &= 16 \end{aligned}$$



$$\begin{aligned} f(0) &= 12(0)^{\frac{2}{3}} - 4(0) \\ f(0) &= 0 \end{aligned}$$

$$\boxed{(8, 16)}$$
  
$$\boxed{(0, 0)}$$