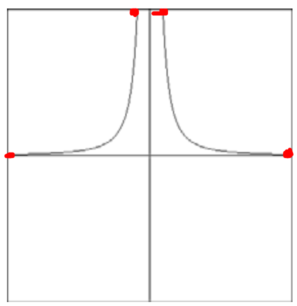
Increasing and Decreasing Functions and the First Derivative Test

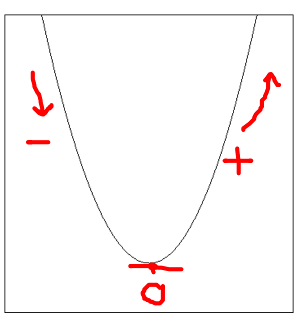
A function is increasing on the interval if $x_1 < x_2$ and $f(x_1) < f(x_2)$.
A function is decreasing on the interval if $x_1 > x_2$ and $f(x_1) > f(x_2)$.

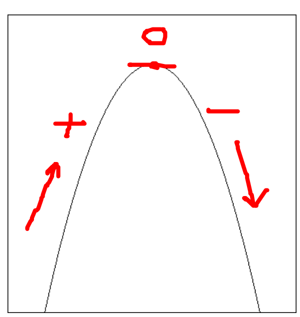


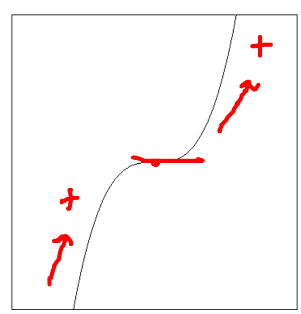
<u>Theorem</u>: Let f be continuous on $[a,b]$ and differentiable on (a,b):
If $f'(x) > 0$ for all x in the interval, then $f(x)$ is increasing on the interval.
If $f'(x) < 0$ for all x in the interval, then $f(x)$ is decreasing on the interval.

<u>First Derivative Test</u>
Let f be continuous on $[a,b]$ and let x_0 be a critical value in the interval.

Relative Mininum	Relative Maximum	Terrace Point
$-\quad 0\quad +$	$+\quad 0\quad -$	$+\quad 0\quad +$ $-\quad 0\quad -$







1. For each function, find the critical numbers of $f(x)$ and determine the intervals for which the function is increasing or decreasing using the first derivative test.

a) $f(x) = x^3 - 9x^2 + 24x - 20$

$f'(x) = 3x^2 - 18x + 24$

$f'(x) = 0$

$\frac{3x^2}{3} - \frac{18x}{3} + \frac{24}{3} = \frac{0}{3}$

$x^2 - 6x + 8 = 0$

$(x-4)(x-2) = 0$ ←

$x - 4 = 0 \qquad x - 2 = 0$

$x = 4 \qquad x = 2$

$f(4) = 4^3 - 9(4)^2 + 24(4) - 20$

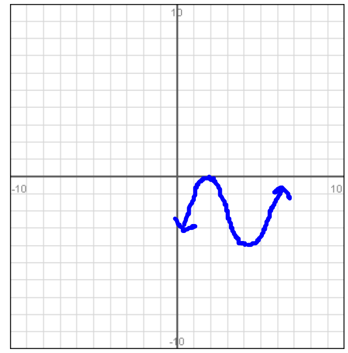
$= 64 - 144 + 96 - 20$

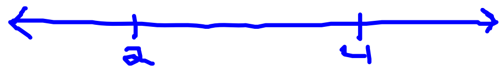
$= -4 \qquad \boxed{(4, -4)}$

$f(2) = 2^3 - 9(2)^2 + 24(2) - 20$

$= 8 - 36 + 48 - 20$

$= 0 \qquad \boxed{(2, 0)}$





$f'(0) = (0-4)(0-2) = (-4)(-2)$

$= 8 \quad \uparrow$

$f'(3) = (3-4)(3-2) = (-1)(1)$

$= -1 \quad \downarrow$

$f'(5) = (5-4)(5-2) = (1)(3)$

$= 3 \quad \uparrow$

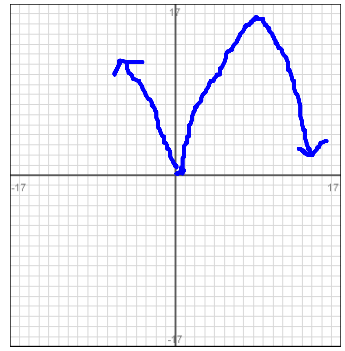
$\boxed{\begin{array}{l} \uparrow \ (-\infty, 2) \cup (4, \infty) \\ \downarrow \ (2, 4) \end{array}}$

b) $f(x) = 12x^{\frac{2}{3}} - 4x$

$f'(x) = \cancel{12}^{4} \cdot \frac{2}{\cancel{3}} x^{-\frac{1}{3}} - 4$

$= \frac{8}{\sqrt[3]{x}} - \frac{4 \cdot \sqrt[3]{x}}{1 \cdot \sqrt[3]{x}}$

LCD $= \sqrt[3]{x}$



$f'(x) = \frac{8 - 4\sqrt[3]{x}}{\sqrt[3]{x}}$

$\underline{f'(x) = 0}$

$8 - 4\sqrt[3]{x} = 0$

$-8 \qquad -8$

$\frac{-4\sqrt[3]{x}}{-4} = \frac{-8}{-4}$

$(\sqrt[3]{x})^3 = (2)^3$

$x = 8$

$f(8) = 12(8)^{2/3} - 4(8)$

$= 12(4) - 32$

$= 48 - 32$

$= 16 \quad \boxed{(8, 16)}$

$\underline{f'(x) = DNE}$

$(\sqrt[3]{x})^3 = (0)^3$

$x = 0$

$f(0) = 12(0)^{2/3} - 4(0)$

$= 0 - 0$

$= 0 \quad \boxed{(0, 0)}$

0   8

$f'(-1) = \frac{8 - 4\sqrt[3]{-1}}{\sqrt[3]{-1}} = \frac{8+4}{-1}$

$= -12 \quad \downarrow$

$f'(1) = \frac{8 - 4\sqrt[3]{1}}{\sqrt[3]{1}} = \frac{8-4}{1}$

$= 4 \quad \uparrow$

$f'(27) = \frac{8 - 4\sqrt[3]{27}}{\sqrt[3]{27}} = \frac{8-12}{3}$

$= -4/3 \quad \downarrow$

$\uparrow \; (0, 8)$

$\downarrow \; (-\infty, 0) \cup (8, \infty)$

c) $f(x) = 3x^4 - 4x^3$

$f'(x) = 12x^3 - 12x^2$

$\underline{f'(x) = 0}$

$12x^3 - 12x^2 = 0$

$12x^2(x-1) = 0$

$\frac{12x^2}{12} = \frac{0}{12}$ $\quad$ $x - 1 = 0$

$\quad\quad\quad\quad\quad\quad\quad\;\; +1 \;\; +1$

$\sqrt{x^2} = \sqrt{0}$ $\quad$ $x = 1$

$x = 0$

$f(0) = 3(0)^4 - 4(0)^3$

$= 0 - 0$

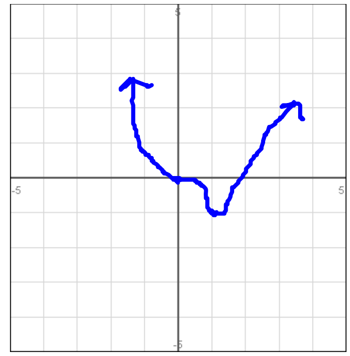
$= 0$ $\quad$ $\boxed{(0,0)}$

$f(1) = 3(1)^4 - 4(1)^3$

$= 3(1) - 4(1)$

$= 3 - 4$

$= -1$ $\quad$ $\boxed{(1,-1)}$



$f'(-1) = 12(-1)^2(-1-1)$

$= 12(1)(-2) = -24$ ↓

$f'(1/2) = 12(1/2)^2(1/2 - 1)$

$= 12(1/4)(-1/2)$

$= -3/2$ ↓

$f'(2) = 12(2)^2(2-1)$

$= 12(4)(1)$

$= 48$ ↑

$$\boxed{\begin{array}{ll} \downarrow & (-\infty, 1) \\ \uparrow & (1, \infty) \end{array}}$$