Curve Sketching (Part 1)

$f(x)$

Domain

x-intercepts set $y=0$

y-intercept set $x=0$

Vertical Asymptote $y=\frac{x+3}{x^2-9}=\frac{\cancel{x+3}}{(\cancel{x+3})(x-3)}$

$x=3$ $y=\frac{1}{x-3}$

hole: $x+3=0$, $x=-3$

VA: $x-3=0$, $x=3$

Hole/Deleted Point

$(-3,-1/6)$ $y=\frac{1}{-3-3}=\frac{1}{-6}$

Horizontal Asymptote

deg num > deg den HA: none

deg num < deg den HA: $y=0$

deg num = deg den HA: $y=$ ratio of leading coeffs.

Slant/Oblique Asymptote

deg num > deg den

use long division

$f'(x)$

Domain

Critical Points

Maximum/Minimum

Increasing/Decreasing

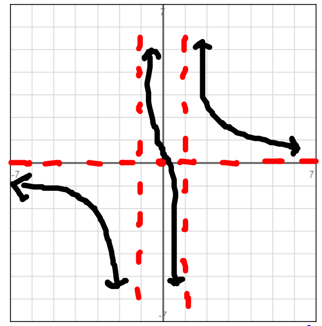
$f''(x)$

Inflection Points

Concave Up/Concave Down

Directions: Sketch the graph of each function.

1. $f(x)=\frac{2x}{x^2-1}$



$\underline{f(x)}$

Domain: $x^2-1=0$

$x^2=1$

$x\neq\pm1$

x-int: $\frac{0}{1}=\frac{2x}{x^2-1}$

$(0,0)$

$2x=0$

$x=0$

$f(0)=\frac{2(0)}{0^2-1}=0$

y-int: $(0,0)$

VA: $x=\pm1$ $\quad y=\frac{2x}{x^2-1}=\frac{2x}{(x+1)(x-1)}$

hole: none

$x+1=0 \qquad x-1=0$

$x=-1 \qquad x=1$

HA: $y=0$

SA: none

deg num = 1

deg den = 2

deg num < deg den

$\underline{f'(x)}$

$$f'(x)=\frac{2(x^2-1)-2x(2x)}{(x^2-1)^2}$$

$$f'(x)=\frac{2x^2-2-4x^2}{(x^2-1)^2}=\frac{-2x^2-2}{(x^2-1)^2}$$

Domain: $x^2-1\neq0$

$x\neq\pm1$

CP: $-2x^2-2=0$

$\frac{-2x^2}{-2}=\frac{2}{-2}$

$x^2=-1$

$x=\pm i$

No CP

$\downarrow \quad -1 \quad \downarrow \quad 1 \quad \downarrow$

$f'(-2)=\frac{-+}{+}=-\ \downarrow$

$f'(0)=\frac{-+}{+}=-\ \downarrow$

$f'(2)=\frac{-+}{+}=-\ \downarrow$

$$f'(x)=\frac{-2x^2-2}{(x^2-1)^2}=\frac{-2(x^2+1)}{(x^2-1)^2}$$

$\underline{f''(x)}$

$$f''(x)=\frac{-4x(x^2-1)^{\cancel{2}}-(-2x^2-2)(2)\cancel{(x^2-1)}(2x)}{(x^2-1)^{\cancel{4}3}}$$

$$f''(x)=\frac{-4x^3+4x-4x(-2x^2-2)}{(x^2-1)^3}=\frac{-4x^3+4x+8x^3+8x}{(x^2-1)^3}$$

$$f''(x)=\frac{4x^3+12x}{(x^2-1)^3}$$

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IP: $4x^3+12x=0$

$4x(x^2+3)=0$

$4x=0 \qquad x^2+3=0$

$x=0 \qquad x^2=-3$

$(0,0) \qquad x=\pm i\sqrt{3}$

$-1 \qquad 0 \qquad 1$

$$f''(x) = \frac{4x(x^2+3)}{(x^2-1)^3}$$

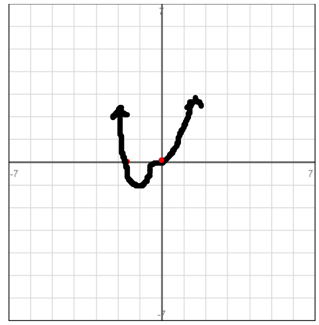
$$f''(-2) = \frac{-\ +}{+} = -$$

$$f''(-1/2) = \frac{-\ +}{-} = +$$

$$f''(1/2) = \frac{+\ +}{-} = -$$

$$f''(2) = \frac{+\ +}{+} = +$$

2. $f(x) = 3x^4 + 4x^3$



Domain: $\mathbb{R}$

x-int: $= 3x^4 + 4x^3$

$(0,0)$ $0 = x^3(3x+4)$

$x^3 = 0$ $3x+4=0$

$(-4/3, 0)$ $x = 0$ $x = -4/3$

y-int: $y = 3(0)^4 + 4(0)^3 = 0$

$(0,0)$

VA
hole
HA
SA
} None

$f'(x)$

$f'(x) = 12x^3 + 12x^2$

Domain: $\mathbb{R}$

CP: $12x^3 + 12x^2 = 0$

$12x^2(x+1) = 0$

$12x^2 = 0$ $x+1=0$

$x = 0$ $x = -1$

$(0,0)$ $(-1,-1)$

minimum Terrace Point

$f(-1) = 3(-1)^4 + 4(-1)^3$

$= 3(1) + 4(-1)$

$= 3 - 4 = -1$

$\downarrow$ -1 $\uparrow$ 0 $\uparrow$

$f'(-2) = + - = -$ $\downarrow$

$f'(-1/2) = + + = +$ $\uparrow$

$f'(1) = + + = +$ $\uparrow$

$f'(x) = 12x^3 + 12x^2 = 12x^2(x+1)$

$f''(x)$

$f''(x) = 36x^2 + 24x$

IP: $36x^2 + 24x = 0$

$12x(3x+2) = 0$

$12x = 0$ $3x+2=0$

$x = 0$ $x = -\frac{2}{3}$

$(0,0)$ $(-2/3, -16/27)$

$f(-\frac{2}{3}) = 3(\frac{-2}{3})^4 + 4(\frac{-2}{3})^3$

$= \cancel{3} \cdot \frac{16}{\cancel{81}\,27} + 4 \cdot \frac{-8}{27} = \frac{16}{27} - \frac{32}{27} = -\frac{16}{27}$

$\cup$ $-\frac{2}{3}$ $\cap$ 0 $\cup$

$f''(-1) = - - = +$ $\cup$

$f''(-\frac{1}{2}) = - + = -$ $\cap$

$f''(1) = + + = +$ $\cup$

$f''(x) = 36x^2 + 24x = 12x(3x+2)$