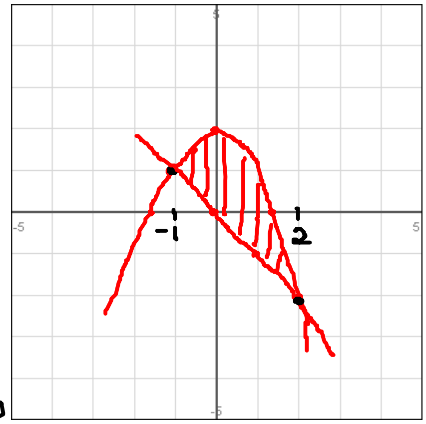
Areas of Regions
Between Curves

$$\text{Area} = \int_a^b f_1(x) - f_2(x)\,dx$$

1. Find the area between the curves $y = 2 - x^2$ and $y = -x$.



$y = -x^2 + 2$
$-x^2 + 2 = 0$
$\sqrt{2} = \sqrt{x^2}$
$\pm\sqrt{2} = x$
$(1.4, 0)(-1.4, 0)$

$y = -x$
$m = -1$
$b = (0,0)$

$-x^2 + 2 = -x$
$0 = x^2 - x - 2$
$0 = (x-2)(x+1)$
$x - 2 = 0 \quad x + 1 = 0$
$x = 2 \quad x = -1$

$$\int_{-1}^{2} (-x^2+2) - (-x)\,dx$$

$$\int_{-1}^{2} -x^2 + 2 + x\,dx = -\frac{x^3}{3} + 2x + \frac{x^2}{2}\Bigg|_{-1}^{2} = \left(\frac{-2^3}{3} + 2(2) + \frac{2^2}{2}\right) - \left(-\frac{(-1)^3}{3} + 2(-1) + \frac{(-1)^2}{2}\right)$$

$$-\frac{8}{3} + 4 + 2 - \left(\frac{1}{3} - 2 + \frac{1}{2}\right) = -\frac{8\cdot 2}{3\cdot 2} + \frac{6\cdot 6}{1\cdot 6} - \frac{1\cdot 2}{3\cdot 2} + \frac{2\cdot 6}{1\cdot 6} - \frac{1\cdot 3}{2\cdot 3} = \frac{-16+36-2+12-3}{6}$$

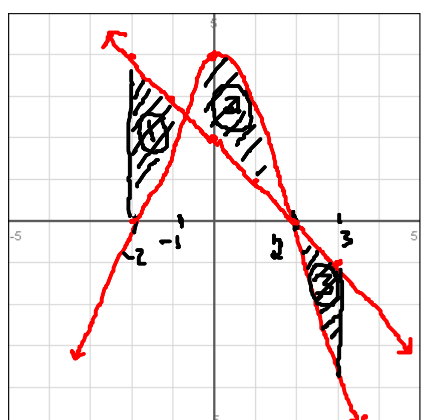
LCD = 6

$$= \boxed{\frac{27}{6}}$$

2. Find the area between the curves $y = 4 - x^2$ and $y = -x + 2$ between $x = -2$ and $x = 3$.

$y = 4 - x^2$
$4 - x^2 = 0$
$4 = x^2$
$\pm 2 = x$
$(2,0)(-2,0)$

$y = -x + 2$
$m = -1$
$b = (0,2)$



$4 - x^2 = -x + 2$
$0 = x^2 - x - 2$
$0 = (x-2)(x+1)$
$x - 2 = 0 \quad x + 1 = 0$
$x = 2 \quad x = -1$

$$\text{1}\ \int_{-2}^{-1} (-x+2) - (4-x^2)\,dx + \text{2}\ \int_{-1}^{2} (4-x^2) - (-x+2)\,dx + \text{3}\ \int_{2}^{3} (-x+2) - (4-x^2)\,dx$$

$$\int_{-2}^{-1} -x + 2 - 4 + x^2\,dx + \int_{-1}^{2} 4 - x^2 + x - 2\,dx + \int_{2}^{3} -x + 2 - 4 + x^2\,dx$$

$$-\frac{x^2}{2}+2x-4x+\frac{x^3}{3}\Bigg|_{-2}^{-1} + 4x-\frac{x^3}{3}+\frac{x^2}{2}-2x\Bigg|_{-1}^{2} + -\frac{x^2}{2}+2x-4x+\frac{x^3}{3}\Bigg|_{2}^{3}$$

$$\left(-\frac{1}{2}-2+4-\frac{1}{3}\right)-\left(-\frac{4}{2}-4+8-\frac{8}{3}\right)+\left(8-\frac{8}{3}+2-4\right)-\left(-4+\frac{1}{3}+\frac{1}{2}+2\right)+\left(-\frac{9}{2}+6-12+\frac{27}{3}\right)$$
$$-\left(-\frac{4}{2}+4-8+\frac{8}{3}\right)$$

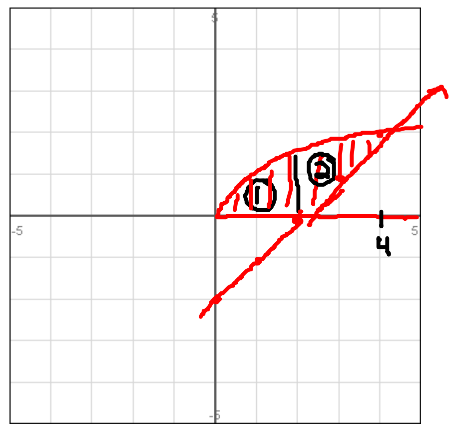
$$=\frac{3\cdot3}{2\cdot3}-\frac{1\cdot2}{3\cdot2}-\frac{2\cdot6}{1\cdot6}+\frac{8\cdot2}{3\cdot2}+\frac{6\cdot6}{1\cdot6}-\frac{8\cdot2}{3\cdot2}+\frac{3\cdot3}{2\cdot3}-\frac{1\cdot2}{3\cdot2}+\frac{3\cdot6}{1\cdot6}-\frac{9\cdot3}{2\cdot3}+\frac{6\cdot6}{1\cdot6}-\frac{8\cdot2}{3\cdot2} \qquad LCD = 6$$

$$=\frac{9-2-12+16+36-16+9-2+18-27+36-16}{6}=\boxed{\frac{49}{6}}$$

3. Find the area between the curves $y=\sqrt{x}$, the $x-axis$ and $y=x-2$ in Quadrant I.

$y=x-2$
$m=1$
$b=(0,-2)$

$(\sqrt{x})^2=(x-2)^2$
$x=(x-2)(x-2)$
$x=x^2-2x-2x+4$
$x=x^2-4x+4$
$0=x^2-5x+4$
$0=(x-1)(x-4)$
$x-1=0 \quad x-4=0$
$x=1 \quad x=4$



1 $\displaystyle\int_0^2 \sqrt{x}-0\,dx$ + 2 $\displaystyle\int_2^4 \sqrt{x}-(x-2)\,dx$

$\sqrt{x} \to x^{\frac12}$

1 $\displaystyle\frac{x^{\frac32}}{\frac32}\Bigg|_0^2$ + 2 $\displaystyle\frac{x^{\frac32}}{\frac32}-\frac{x^2}{2}+2x\Bigg|_2^4 = \left(\frac{2}{3}\cdot2^{\frac32}-\cancel{\frac{2}{3}\cdot0^{3/2}}\right)+\left(\frac{2}{3}\cdot4^{\frac32}-\frac{16}{2}+8\right)$
$$-\left(\frac{2}{3}\cdot2^{\frac32}-\frac{4}{2}+4\right)$$

$$=\cancel{\frac{2}{3}\sqrt{8}}+\frac{2}{3}\cdot8\cancel{-8}\cancel{+8}-\cancel{\frac{2}{3}\sqrt{8}}+2-4=\frac{16}{3}-\frac{2\cdot3}{1\cdot3}=\frac{16-6}{3}=\boxed{\frac{10}{3}}$$

4. Find the area between the curves $f(y) = y(2-y)$ and $g(y) = -y$.

$x = y(2-y)$

$x = 2y - y^2$

$y = 2x - x^2$

$x = \frac{-b}{2a} = \frac{-2}{2(-1)} = 1$

$y = 2(1) - 1^2 = 1$

vertex $(1,1)$

$x = -y$

$y = -x$

$m = -1$

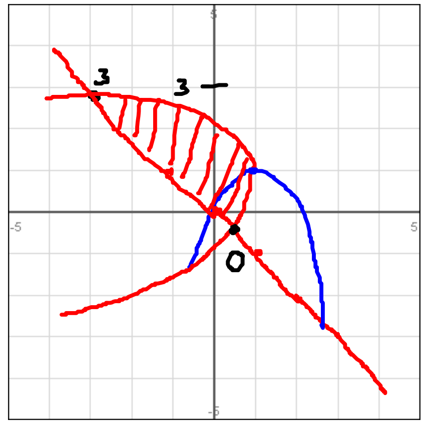
$b = (0,0)$

$2y - y^2 = -y$

$0 = y^2 - 3y$

$0 = y(y-3)$

$y=0 \quad y=3$



$$\int_0^3 y(2-y) - (-y)\, dy$$

$$\int_0^3 2y - y^2 + y\, dy = \int_0^3 -y^2 + 3y\, dy = \frac{-y^3}{3} + \frac{3y^2}{2}\Big|_0^3$$

$$= \left(-\frac{27}{3} + \frac{27}{2}\right) - \left(\frac{0}{3} + \frac{0}{2}\right) = \frac{-27 \cdot 2}{3 \cdot 2} + \frac{27 \cdot 3}{2 \cdot 3} = \frac{-54 + 81}{6} = \frac{27}{6} = \boxed{\frac{9}{2}}$$

LCD = 6