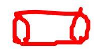
Finding Volumes of Solids Using the Shell Method

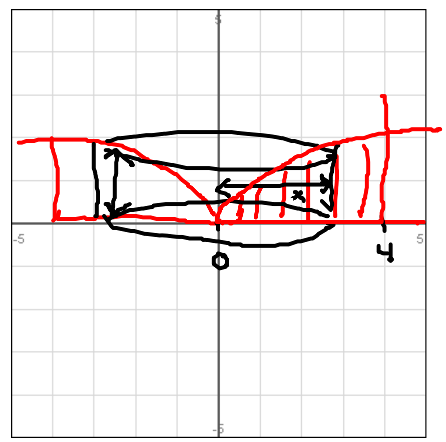
Vertical Shell: Volume $= \int_a^b 2\pi \cdot (\text{Shell Radius}) \cdot (\text{Shell Height})\, dx = \int_a^b 2\pi \cdot x \cdot f(x)\, dx$ 

Horizontal Shell: Volume $= \int_c^d 2\pi \cdot (\text{Shell Radius}) \cdot (\text{Shell Height})\, dy = \int_c^d 2\pi \cdot y \cdot f(y)\, dy$ 

1. The region bounded by the curve $y = \sqrt{x}$, the $x - axis$ and the line $x = 4$ is revolved about the line $y - axis$ to generate a solid. Find the volume.

$$\int_a^b 2\pi x\, f(x)\, dx$$

$f(x) =$ height
$= \sqrt{x} - 0$
$= \sqrt{x}$




$$\int_0^4 2\pi x \sqrt{x}\, dx$$

$$\int_0^4 2\pi x \cdot x^{\frac{1}{2}}\, dx = \int_0^4 2\pi x^{3/2}\, dx$$

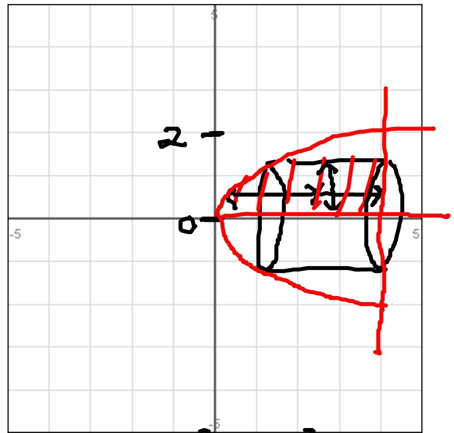
$$2\pi \int_0^4 x^{3/2}\, dx = 2\pi \left. \frac{x^{5/2}}{\frac{5}{2}} \right|_0^4 = 2\pi \cdot \frac{2}{5} x^{5/2} \Big|_0^4 = \frac{4\pi}{5} x^{5/2} \Big|_0^4$$

$$\frac{4\pi}{5}\left(4^{5/2} - 0^{5/2}\right) = \frac{4\pi}{5}(32 - 0) = \frac{4\pi}{5} \cdot 32 = \boxed{\frac{128\pi}{5}}$$

2. The region bounded by the curve $y = \sqrt{x}$, the $x-axis$ and the line $x = 4$ is revolved about the $x-axis$ to generate a solid. Find the volume.

$$\int_c^d 2\pi y\, f(y)\, dy$$

height $= f(y)$
$= 4 - y^2$




$(y)^2 = (\sqrt{x})^2$
$x = y^2$

$y = \sqrt{x}$
$y = \sqrt{4}$
$y = 2$

$$\int_0^2 2\pi y(4-y^2)\,dx = \int_0^2 2\pi(4y-y^3)\,dy$$

$$2\pi \int_0^2 4y - y^3 \,dy$$

$$2\pi\left[\frac{4y^2}{2} - \frac{y^4}{4}\right]\Bigg|_0^2 = 2\pi\left[2y^2 - \frac{y^4}{4}\right]_0^2$$

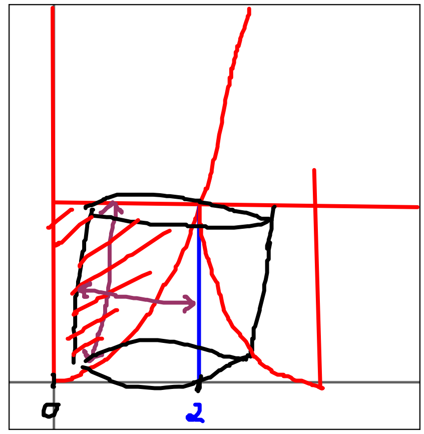
$$2\pi\left[\left(8 - \frac{16}{4}\right) - (0-0)\right] = 2\pi(8-4) = 2\pi(4) = \boxed{8\pi}$$

3. The region in the first quadrant bounded by the parabola $y = x^2$, the $y-axis$, and the line $y = 4$ is revolved about the line $x = 2$ to generate a solid. Find the volume.

$y = 2^2$

$y = 4$

$(2,4)$




$$\int_a^b 2\pi x\, f(x)\, dx$$

radius height

height = $f(x)$
$= 4 - x^2$

Radius = $2 - x$

$$\int_0^2 2\pi(2-x)(4-x^2)\,dx$$

$8 - 2x^2 - 4x + x^3$

$x^3 - 2x^2 - 4x + 8$

$$2\pi\int_0^2 x^3 - 2x^2 - 4x + 8\,dx = 2\pi\left[\frac{x^4}{4} - \frac{2x^3}{3} - \frac{4x^2}{2} + 8x\right]_0^2$$

$$2\pi\left[\left(\frac{16}{4} - \frac{16}{3} - 8 + 16\right) - (0)\right] = 2\pi\left[4 - \frac{16}{3} - 8 + 16\right]$$

LCD = 3

$$2\pi\left[\frac{3\cdot 12}{3\cdot 1} - \frac{16}{3}\right] = 2\pi\left[\frac{36-16}{3}\right] = 2\pi\cdot\frac{20}{3} = \boxed{\frac{40\pi}{3}}$$