Exponential Growth and Decay

Compound Interest Formulas

$$A = P\left(1+\frac{r}{n}\right)^{nt}$$ * n compoundings per year

$$A = Pe^{rt}$$ * continuous compounding

P → initial amount
A → future amount
r → interest rate (as a decimal)
n → number of compoundings
t → time in years

Exponential Growth and Decay

$$y = Ce^{kt}$$

C → initial amount
y → future amount
t → time
K < rate of growth, K>0; rate of decay, K<0

1. If an initial investment of \$2,500 is compounded continuously at a rate of 8.5%, how long would it take for your investment to double?

$A = Pe^{Rt}$

P = 2500
R = 8.5% = .085
A = 5000
t = ?

$$\frac{5000}{2500} = \frac{2500}{2500}e^{.085t}$$
$$2 = e^{.085t}$$
$$\ln 2 = \ln e^{.085t}$$
$$\ln 2 = .085t \ln e$$
$$\frac{\ln 2}{.085} = \frac{.085t}{.085}$$
$$\boxed{t = 8.15 \text{ years}}$$

2. Which option is better for an initial investment of $3,000?

   a) compounded continuously at 7% for 10 years

   b) compounded daily at 8% for 10 years

a) $A = Pe^{Rt}$

$P = 3000$

$R = 7\% = .07$

$t = 10$

$A = 3000e^{.07 \times 10}$

$A = 3000e^{.7}$

$A = \$6041.26$

$\boxed{\text{Option B}}$

b) $A = P\left(1 + \frac{R}{N}\right)^{Nt}$

$P = 3000$

$R = 8\% = .08$

$t = 10$

$N = 365$

$A = 3000\left(1 + \frac{.08}{365}\right)^{365 \times 10}$

$A = 3000\left(1 + \frac{.08}{365}\right)^{3650}$

$A = \$6676.04$

3. The population of a city is increasing exponentially. In 1990 there were 25,000 people and in 2008 there were 36,000 people.

a) Find the initial population if the city was founded in 1950.

b) When will the population reach 75,000?

a) $y = Ce^{kt}$

$t = 40,\ y = 25{,}000$

$t = 58,\ y = 36{,}000$

$C = \dfrac{36{,}000}{e^{58k}}$

$k = .020258$

$C = \dfrac{36{,}000}{e^{58 \times .020258}}$

$\boxed{C = 11{,}118 \text{ people}}$

$25{,}000 = Ce^{40k}$

$36{,}000 = Ce^{58k} \rightarrow C = \dfrac{36{,}000}{e^{58k}}$

$25{,}000 = \dfrac{36{,}000}{e^{58k}} e^{40k}$

$\dfrac{25{,}000}{36{,}000} = \dfrac{36000 e^{-18k}}{36{,}000}$

$\dfrac{25}{36} = e^{-18k}$

$\ln\left(\dfrac{25}{36}\right) = \ln e^{-18k}$

$\ln\left(\dfrac{25}{36}\right) = -18k \ln e$

$\dfrac{\ln\left(\frac{25}{36}\right)}{-18} = \dfrac{-18k}{-18}$

$k = .020258$

b) $y = Ce^{kt}$

$y = 75{,}000$

$C = 11{,}118$

$k = .020258$

$t = ?$

$\dfrac{75{,}000}{11{,}118} = \dfrac{11{,}118 e^{.020258t}}{11{,}118}$

$6.74582 = e^{.020258t}$

$\ln 6.74582 = \ln e^{.020258t}$

$\ln 6.74582 = .020258t \ln e$

$\dfrac{\ln 6.74582}{.020258} = \dfrac{.020258t}{.020258}$

$\boxed{\begin{array}{l} t = 94.23 \text{ years} \\ \text{in the year } 2044 \end{array}}$

4. Carbon-14 has a half-life of 5,715 years. If the initial quantity is 100 grams, how much will remain after 200 years?

$y = Ce^{Kt}$
$C = 100$
$y = 50$
$t = 5715$
$K = ?$

$\frac{50}{100} = \frac{100e^{5715K}}{100}$

$\frac{1}{2} = e^{5715K}$

$\ln(\frac{1}{2}) = \ln e^{5715K}$

$\ln(\frac{1}{2}) = 5715K \cdot \ln e$

$\frac{\ln(\frac{1}{2})}{5715} = \frac{5715K}{5715}$

$K = -.000121$

$y = Ce^{Kt}$
$C = 100$
$K = -.000121$
$t = 200$

$y = 100e^{-.000121 \times 200}$

$\boxed{y = 97.6 \text{ grams}}$

5. Carbon-14 has a half-life of 5,715 years. What percent will remain after 1,000 years?

$y = Ce^{Kt}$

$t = 5715$ when $\frac{y}{c} = \frac{1}{2}$

$\frac{1}{2} = e^{5715K}$

$\ln(\frac{1}{2}) = \ln e^{5715K}$

$\ln(\frac{1}{2}) = 5715K \cdot \ln e$

$\frac{\ln(\frac{1}{2})}{5715} = \frac{5715K}{5715}$

$K = -.000121$

$y = Ce^{Kt}$
$K = -.000121$
$t = 1000$

$\frac{y}{c} = ?$

$\frac{y}{c} = e^{-.000121 \times 1000}$

$\frac{y}{c} = .886$

$\boxed{88.6\%}$

6. Carbon-14 has a half-life of 5,715 years. When will 25% of carbon remain?

$y = Ce^{kt}$

$t = 5715$ when $\frac{y}{c} = \frac{1}{2}$

$\frac{1}{2} = e^{5715k}$

$\ln(\frac{1}{2}) = \ln e^{5715k}$

$\ln(\frac{1}{2}) = 5715k \cdot \ln e$

$\frac{\ln(\frac{1}{2})}{5715} = \frac{5715k}{5715}$

$k = -.000121$

$y = Ce^{kt}$

$k = -.000121$

$\frac{y}{c} = 25\% = .25$

$t = ?$

$.25 = e^{-.000121t}$

$\ln(.25) = \ln e^{-.000121t}$

$\ln(.25) = -.000121t \cdot \ln e$

$\frac{\ln(.25)}{-.000121} = \frac{-.000121t}{-.000121}$

$\boxed{t = 11{,}430 \text{ years}}$