Algebraic Proofs

<u>Algebraic Properties of Equality</u>

Addition Property - If $a = b$, then $a + c = b + c$.

Subtraction Property - If $a = b$, then $a - c = b - c$.

Multiplication Property - If $a = b$, then $a \cdot c = b \cdot c$

Division Property - If $a = b$ and $c \neq 0$, then $a \div c = b \div c$

Distributive Property - For all numbers a, b and c, $a(b + c) = a \cdot b + a \cdot c$.

Reflexive Property - For any real number a, $a = a$.

Symmetric Property - If $a = b$, then $b = a$.

Transitive Property - If $a = b$ and $b = c$, then $a = c$.

Substitution Property - If $a = b$, then a can be substituted for b in any equation or expression.

<u>Properties of Equality for Segment Length and Angle Measure</u>

	<u>Segment Length</u>	<u>Angle Measure</u>
Reflexive -	$AB = AB$	$m\angle A = m\angle A$
Symmetric -	If $AB = CD$, then $CD = AB$.	If $m\angle A = m\angle B$, then $m\angle B = m\angle A$.
Transitive -	If $AB = CD$ and $CD = EF$, then $AB = EF$.	If $m\angle A = m\angle B$ and $m\angle B = m\angle C$, then $m\angle A = m\angle C$.

Directions: Name the property of equality that justifies each statement.

1. If $LM = NO$ and $NO = PQ$, then $LM = PQ$.

> Transitive Property

2. If $m\angle A = 10°$, then $12° + m\angle A = \underline{22°}$.

> $12° + 10° = 22°$
> $22° = 22°$
> Addition Property of Equality

3. If $AB = 5$ and $AC = AB + 8$ then $AC = 13$.

$AC = 5 + 8$

$AC = 13$

Substitution Property

4. If $m\angle C = m\angle D$, then $m\angle D = m\angle C$.

Symmetric Property

5. If $5(m\angle A) = 90°$, then $m\angle A = 18°$.

$\dfrac{}{5} \qquad \dfrac{}{5}$

$m\angle A = 18°$

Division Property

Directions: Write an algebraic proof for each.

6. Given: $5x - 18 = 3x + 2$
   Prove: $x = 10$

$*\ 5x - 18 = 3x + 2$
$\quad -3x \qquad -3x$
$*\ 2x - 18 = 2$
$\qquad +18 \quad +18$
$*\ \dfrac{2x}{2} = \dfrac{20}{2}$
$*\quad x = 10$

Statements	Reason
1) $5x - 18 = 3x + 2$	1) Given
2) $2x - 18 = 2$	2) Subtract. Prop. of Equality
3) $2x = 20$	3) Addit. Prop. of Equality
4) $x = 10$	4) Division Prop. of Equality.

7. Given: $2(3x+1) = 4x+8$

Prove: $x = 3$

$* \quad 2(3x+1) = 4x+8$

$* \quad 6x + 2 = 4x + 8$

$\quad -4x \qquad -4x$

$* \quad 2x + 2 = 8$

$\qquad -2 \quad -2$

$* \quad 2x = 6$

$\qquad \overline{2} \quad \overline{2}$

$* \quad x = 3$

Statements	Reason
1) $2(3x+1) = 4x+8$	1) Given
2) $6x+2 = 4x+8$	2) Distributive Prop.
3) $2x+2 = 8$	3) Subt. Prop. of Equal.
4) $2x = 6$	4) Subt. Prop. of Equal.
5) $x = 3$	5) Div. Prop. of Equal.

8. Given: $5x - 3(9x+12) = 8$

Prove: $x = -2$

$* \quad 5x - 3(9x+12) = 8$

$* \quad 5x - 27x - 36 = 8$

$* \quad -22x - 36 = 8$

$\qquad +36 \quad +36$

$* \quad -22x = 44$

$\qquad \overline{-22} \quad \overline{-22}$

$* \quad x = -2$

Statement	Reason
1) $5x-3(9x+12) = 8$	1) Given
2) $5x-27x-36 = 8$	2) Distributive Property
3) $-22x-36 = 8$	3) Simplify
4) $-22x = 44$	4) Addition Prop. of Equality
5) $x = -2$	5) Division Prop. of Equality