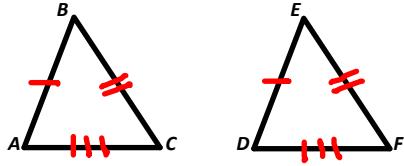


Proving Congruent Triangles

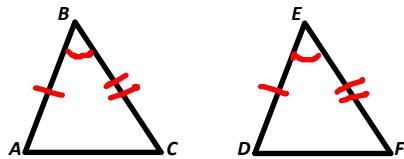
Side-Side-Side Postulate (SSS) - If the sides of one triangle are congruent to the sides of a second triangle, then the triangles are congruent.



$$\Delta ABC \cong \Delta DEF$$

sss Post.

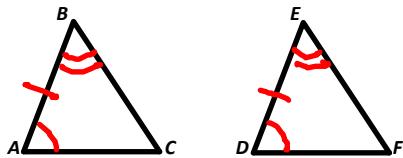
Side-Angle-Side Postulate (SAS) - If two sides and the included angle of one triangle are congruent to two sides and the included angle of another triangle, then the triangles are congruent.



$$\Delta ABC \cong \Delta DEF$$

SAS Post.

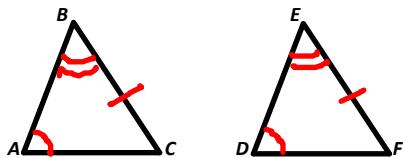
Angle-Side-Angle Postulate (ASA) - If two angles and the included side of one triangle are congruent to two angles and the included side of another triangle, then the triangles are congruent.



$$\Delta ABC \cong \Delta DEF$$

ASA Post.

Angle-Angle-Side Theorem (AAS) - If two angles and a nonincluded side of one triangle are congruent to the corresponding two angles and side of a second triangle, then the triangles are congruent.



$$\Delta ABC \cong \Delta DEF$$

AAS Theorem

C.P.C.T.C. - Corresponding Parts of Congruent Triangles are Congruent

Directions: Write a proof for each.

1. Given: $\angle D \cong \angle E$

C is the midpoint of \overline{DE}

Prove: $\triangle DBC \cong \triangle EAC$ •

SAS, SSS, ASA, AAS

Statement

1. $\angle D \cong \angle E$

C is the midpoint of \overline{DE}

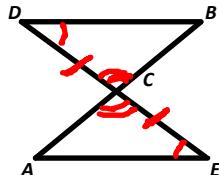
$$2) \overline{DC} \cong \overline{EC}$$

$$3) \angle DCB \cong \angle ECA$$

$$4) \triangle DBC \cong \triangle EAC$$

Reason

1. Given



2) Def. of midpoint

3) Vertical \angle Theorem

4) ASA Post.

2. Given: \overline{DE} bisects \overline{BA} •

\overline{BA} bisects \overline{DE} •

Prove: $\triangle DBC \cong \triangle EAC$ •

Statement

1. \overline{DE} bisects \overline{BA}

\overline{BA} bisects \overline{DE}

$$2) \overline{BC} \cong \overline{AC}$$

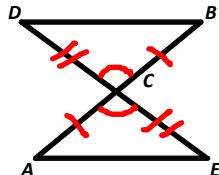
$$\overline{DC} \cong \overline{EC}$$

$$3) \angle DCB \cong \angle ECA$$

$$4) \triangle DBC \cong \triangle EAC$$

Reason

1. Given



2) Def. of Bisect

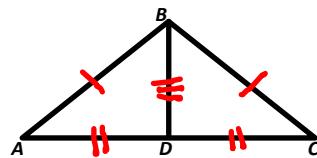
3) Vertical \angle Theorem

4) SAS Post.

3. Given: $\triangle ABC$ is an isosceles triangle with vertex $\angle ABC$.

D is the midpoint of \overline{AC} .

Prove: $\triangle ABD \cong \triangle CBD$.



Statement

1. $\triangle ABC$ is an isosceles triangle with vertex $\angle ABC$

D is the midpoint of \overline{AC}

Reason

1. Given

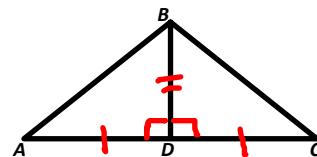
- 2) $\overline{AB} \cong \overline{CB}$
3) $\overline{AD} \cong \overline{CD}$
4) $\overline{BD} \cong \overline{BD}$
5) $\triangle ABD \cong \triangle CBD$

- 2) Def. of Isosceles \triangle
3) Def. of midpoint
4) Reflexive
5) SSS Post.

4. Given: $\overline{BD} \perp \overline{AC}$.

\overline{BD} bisects \overline{AC} .

Prove: $\triangle ABD \cong \triangle CBD$.



Statement

1. $\overline{BD} \perp \overline{AC}$

\overline{BD} bisects \overline{AC}

Reason

1. Given

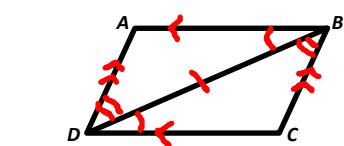
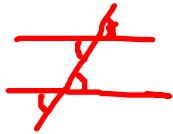
- 2) $m\angle BDA = 90^\circ$
 $m\angle BDC = 90^\circ$
3) $m\angle BDA = m\angle BDC$
4) $\angle BDA \cong \angle BDC$
5) $\overline{AD} \cong \overline{CD}$
6) $\overline{BD} \cong \overline{BD}$
7) $\triangle ABD \cong \triangle CBD$

- 2) Def. of Perpendicular
3) Substitution
4) Def. of \cong angles
5) Def. of bisect
6) Reflexive
7) SAS Post.

5. Given: $\overline{AB} \parallel \overline{DC}$.

$$\overline{BC} \parallel \overline{AD}$$

Prove: $\angle A \cong \angle C$.



Statement

1. $\overline{AB} \parallel \overline{DC}$

$$\overline{BC} \parallel \overline{AD}$$

2) $\angle ABD \cong \angle CBD$

$$\angle ADB \cong \angle CBD$$

3) $\overline{BD} \cong \overline{DB}$

4) $\triangle ABD \cong \triangle CBD$

5) $\angle A \cong \angle C$

Reason

1. Given

2) Alternate Interior \angle
Theorem

3) Reflexive

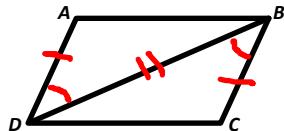
4) ASA Post.

5) CPCTC

6. Given: $\angle ADB \cong \angle CBD$.

$$\overline{DA} \cong \overline{BC}$$

Prove: $\overline{AB} \cong \overline{CD}$



Statement

1. $\angle ADB \cong \angle CBD$

$$\overline{DA} \cong \overline{BC}$$

2) $\overline{DB} \cong \overline{BD}$

3) $\triangle ADB \cong \triangle CBD$

4) $\overline{AB} \cong \overline{CD}$

Reason

1. Given

2) Reflexive

3) SAS Post.

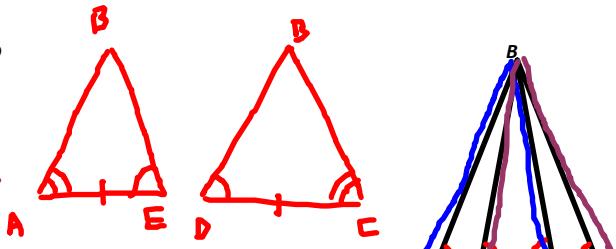
4) CPCTC

7. Given: $\angle BDE \cong \angle BED$

$$\angle A \cong \angle C$$

$$\overline{AD} \cong \overline{CE}$$

Prove: $\triangle BAE \cong \triangle BCD$



Statement

1. $\angle BDE \cong \angle BED$ •

$$\angle A \cong \angle C$$
 •

$$\overline{AD} \cong \overline{CE}$$
 •

- 2) $AD = CE$ ←
- 3) $\Sigma D \cong \Sigma D$
- 4) $D \cong E$
- 5) $AE = \overline{AD} + \overline{DE}$ ←
- 6) $CD = \overline{CE} + \overline{ED}$ *
- 7) $CD = AE$
- 8) $\overline{CD} \cong \overline{AE}$
- 9) $\triangle BAE \cong \triangle BCD$

8. Given: $\angle ABE \cong \angle CBD$ -

$$\angle A \cong \angle C$$
 :

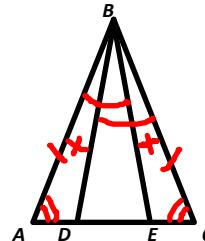
$$\overline{AB} \cong \overline{CB}$$
 :

Prove: $\triangle ABD \cong \triangle CBE$

Reason

1. Given

- 2) Def. of \cong segments
- 3) Reflexive
- 4) Def of \cong segments
- 5) Segment Addition Post.
- 6) Substitution
- 7) Substitution
- 8) Def of \cong segments
- 9) ASA Post.



Statement

1. $\angle ABE \cong \angle CBD$

$$\angle A \cong \angle C$$

$$\overline{AB} \cong \overline{CB}$$

- 2) $m\angle ABE = m\angle CBD$ ←
- 3) $\angle DBE \cong \angle EBD$
- 4) $m\angle DBE = m\angle EBD$
- 5) $m\angle ABD = m\angle ABE + m\angle DBE$
- * $m\angle CBD = m\angle CBE + m\angle EBD$
- * 6) $m\angle CBD = m\angle ABD + m\angle DBE$
- 7) $m\angle CBE + m\angle EBD = m\angle ABD + m\angle DBE$ 7) Substitution
- 8) $m\angle CBE = m\angle ABD$
- 9) $\angle CBE \cong \angle ABD$
- 10) $\triangle ABD \cong \triangle CBE$

Reason

1. Given

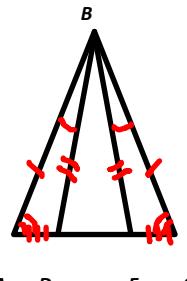
- 2) Def of \cong angles
- 3) Reflexive
- 4) Def of \cong angles
- 5) Angle Addition Post.
- 6) Substitution
- 7) Substitution
- 8) Subtraction
- 9) Def. of \cong angles
- 10) ASA Post.

9. Given: $\angle ABD \cong \angle CBE$

$$\angle A \cong \angle C$$

$$\overline{AB} \cong \overline{CB}$$

Prove: $\overline{AE} \cong \overline{CD}$



Statement

$$1. \angle ABD \cong \angle CBE$$

$$\angle A \cong \angle C$$

$$\overline{AB} \cong \overline{CB}$$

Reason

$$1. \text{ Given}$$

- 2) $\triangle ABD \cong \triangle CBE$
- 3) $\overline{AD} \cong \overline{CE}$
- 4) $AD = CE$ ←
- 5) $\overline{DE} \cong \overline{ED}$ ←
- 6) $DE = ED$
- 7) $AE = AD + DE$ ←
 $CD = CE + ED$ *
- 8) $AE = CE + DE$ *
- 9) $CD = AE$
- 10) $\overline{CD} \cong \overline{AE}$
- 11) $\overline{AB} \cong \overline{CD}$

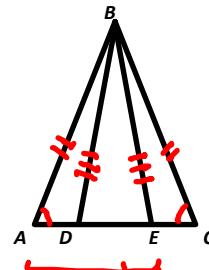
- 2) ASA Post.
- 3) CPCTC
- 4) Def. of \cong segments
- 5) Reflexive
- 6) Def. of \cong segments
- 7) Segment Addition Post
- 8) Substitution
- 9) Substitution
- 10) Def. of \cong segments
- 11) Symmetric

10. Given: $\overline{AE} \cong \overline{CD}$.

$$\angle A \cong \angle C$$

$$\overline{AB} \cong \overline{CB}$$

Prove: $\overline{DB} \cong \overline{EB}$



Statement

$$1. \overline{AE} \cong \overline{CD}$$

$$\angle A \cong \angle C$$

$$\overline{AB} \cong \overline{CB}$$

- 2) $\triangle BAE \cong \triangle BCD$
- 3) $\overline{DE} \cong \overline{EB}$

Reason

$$1. \text{ Given}$$

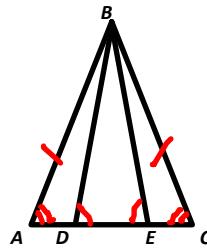
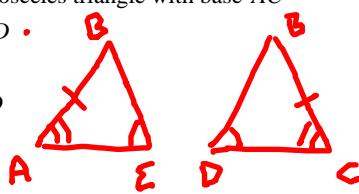
- 2) SAS Post
- 3) CPCTC

11. Given: $\triangle ABC$ is an isosceles triangle with base \overline{AC}

$$\angle BDE \cong \angle BED \cdot$$

$$\angle A \cong \angle C$$

Prove: $\triangle ABE \cong \triangle CBD$



Statement

1. $\triangle ABC$ is an isosceles triangle with base \overline{AC}

$$\angle BDE \cong \angle BED$$

$$\angle A \cong \angle C$$

$$2) \overline{AB} \cong \overline{CB}$$

$$3) \triangle ABE \cong \triangle CBD$$

Reason

1. Given

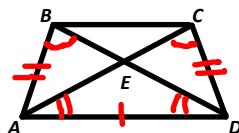
2) Def. of Isos. \triangle

3) AAS Theorem

12. Given: $\angle ABE \cong \angle DCE$ •

$$\angle EAD \cong \angle EDA$$

Prove: $\overline{BA} \cong \overline{CD}$ •



Statement

1. $\angle ABE \cong \angle DCE$

$$\angle EAD \cong \angle EDA$$

$$2) \overline{AD} \cong \overline{DA}$$

$$3) \triangle BAD \cong \triangle CDA$$

$$4) \overline{BA} \cong \overline{CD}$$

Reason

1. Given

2) Reflexive

3) AAS Theorem

4) CPCTC

13. Given: $\angle A \cong \angle ABE$.

$$\angle ECD \cong \angle D \cdot \leftarrow$$

$$\angle A \cong \angle D \cdot \leftarrow$$

$$\overline{AE} \cong \overline{DE} .$$

Prove: $\triangle BEC$ is an isosceles triangle

Statement

1. $\angle A \cong \angle ABE$

$$\angle ECD \cong \angle D$$

$$\angle A \cong \angle D$$

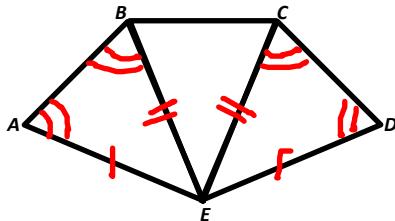
$$\overline{AE} \cong \overline{DE}$$

2) $\angle ECD \cong \angle A$

3) $\triangle BAE \cong \triangle CDE$

4) $\overline{BE} \cong \overline{CE}$

5) $\triangle BEC$ is isos. \triangle



Reason

1. Given

2) Transitive

3) AAS Theorem

4) CPCTC

5) Def of isos. \triangle