Geometric Mean and Similar Triangles

Geometric Mean - of two positive numbers a and b is the positive number x such that $\frac{x}{a}=\frac{b}{x}$.

1. Find the geometric mean of the two numbers.

a) 9 and 4

$\frac{x}{9}=\frac{4}{x}$

$\sqrt{x^2}=\sqrt{36}$

$\boxed{x=6}$

b) 7 and 28

$\frac{x}{7}=\frac{28}{x}$

$\sqrt{x^2}=\sqrt{196}$

$\boxed{x=14}$

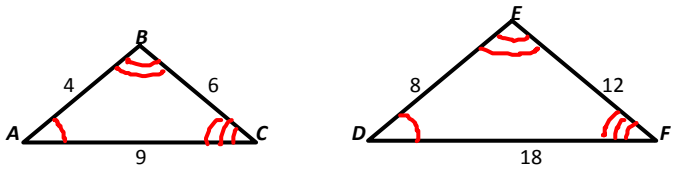
c) 5 and 16

$\frac{x}{5}=\frac{16}{x}$

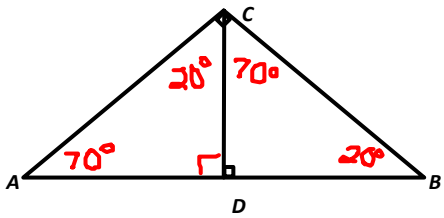
$\sqrt{x^2}=\sqrt{80}$

$\boxed{x=4\sqrt{5}}$

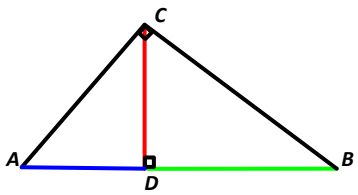
Similar Triangles - Two triangles are similar if two of their corresponding angles are congruent.



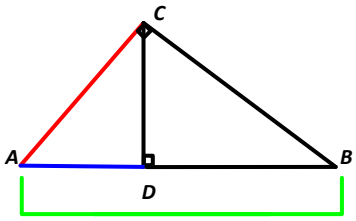
If $\triangle ABC$ is a right triangle and $\overline{CD}$ is the altitude, then $\triangle ABC \sim \triangle CBD \sim \triangle ACD$.



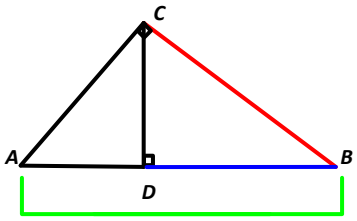
Using the Geometric Mean to Solve Triangles



$\frac{CD}{AD}=\frac{DB}{CD}$



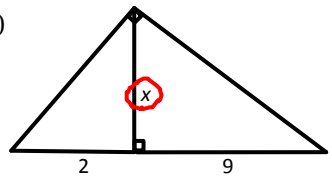
$\frac{AC}{AD}=\frac{AB}{AC}$



$\frac{CB}{BD}=\frac{BA}{CB}$

2. Find the value of each variable.

a)



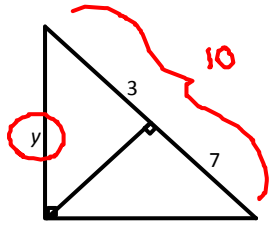
$\frac{x}{2} = \frac{9}{x}$

$\sqrt{x^2} = \sqrt{18}$

$x = \sqrt{18} < \begin{matrix} 9 \\ 2 \end{matrix}$

$\boxed{x = 3\sqrt{2}}$

b)

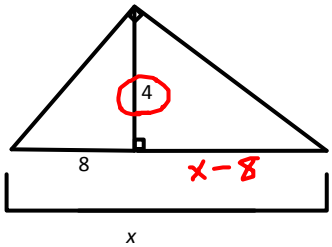


$\frac{y}{3} = \frac{10}{y}$

$\sqrt{y^2} = \sqrt{30}$

$\boxed{y = \sqrt{30}}$

c)



$\frac{4}{8} = \frac{x-8}{4}$

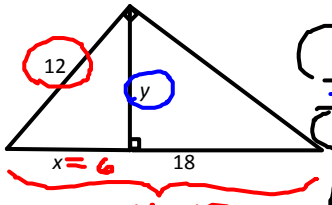
$8(x-8) = 16$

$8x - 64 = 16$

$+64 \quad +64$

$\frac{8x}{8} = \frac{80}{8}$ $\boxed{x = 10}$

d)



$\frac{12}{x} = \frac{x+18}{12}$

$x(x+18) = 144$

$x^2 + 18x = 144$

$-144 \quad -144$

$x^2 + 18x - 144 = 0$

$(x-6)(x+24) = 0$

$x - 6 = 0$ $\quad$ $x + 24 = 0$

$\boxed{x = 6}$ $\quad$ $\cancel{x = -24}$

$\frac{y}{6} = \frac{18}{y}$

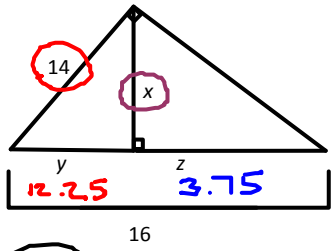
$\sqrt{y^2} = \sqrt{108}$

$y = \sqrt{108}$

$y = \sqrt{36 \cdot 3}$

$\boxed{y = 6\sqrt{3}}$

e)



$$\frac{14}{y} = \frac{16}{14}$$

$$\frac{16y}{16} = \frac{196}{16}$$

$$\boxed{y = 12.25}$$

$$12.25 + z = 16$$
$$-12.25 \quad -12.25$$

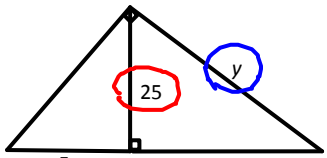
$$\boxed{z = 3.75}$$

$$\frac{x}{12.25} = \frac{3.75}{x}$$

$$\sqrt{x^2} = \sqrt{45.9375}$$

$$\boxed{x = 6.78}$$

f)



$25\sqrt{5}$ $\quad$ $5\sqrt{5}$

$$\frac{25}{5x} = \frac{x}{25}$$

$$\frac{5x^2}{5} = \frac{625}{5}$$

$$\sqrt{x^2} = \sqrt{125}$$

$$x = \sqrt{125}$$

$$x = \sqrt{25 \cdot 5}$$

$$\boxed{x = 5\sqrt{5}}$$

$$\frac{y}{5\sqrt{5}} = \frac{30\sqrt{5}}{y}$$

$$y^2 = 5\sqrt{5} \cdot 30\sqrt{5}$$

$$y^2 = 150\sqrt{25}$$

$$y^2 = 150 \cdot 5$$

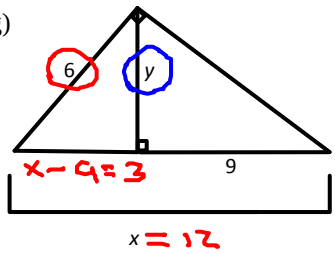
$$\sqrt{y^2} = \sqrt{750}$$

$$y = \sqrt{750}$$

$$y = \sqrt{25 \cdot 30}$$

$$\boxed{y = 5\sqrt{30}}$$

g)



$$\frac{6}{x-9} = \frac{x}{6}$$

$$x(x-9) = 36$$

$$x^2 - 9x = 36$$
$$-36 \quad -36$$

$$x^2 - 9x - 36 = 0$$

$$(x-12)(x+3) = 0$$

$$x - 12 = 0 \qquad x + 3 = 0$$

$$\boxed{x = 12} \qquad \cancel{x = -3}$$

$$\frac{y}{3} = \frac{9}{y}$$

$$\sqrt{y^2} = \sqrt{27}$$

$$y = \sqrt{27}$$

$$y = \sqrt{9 \cdot 3}$$

$$\boxed{y = 3\sqrt{3}}$$