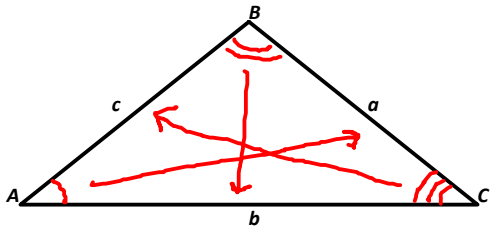
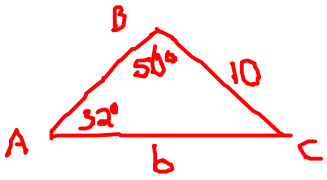
The Law of Sines

The Law of Sines is used to solve oblique triangles (triangles that do not have a right angle) when you have an angle-side pair.



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

1. In $\triangle ABC$, $a = 10$, $m\angle A = 32^\circ$, and $m\angle B = 50^\circ$. Find b.



$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

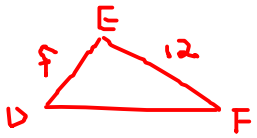
$$\frac{\sin 32}{10} = \frac{\sin 50}{b}$$

$$b \cdot \frac{\sin 32}{\sin 32} = \frac{10 \cdot \sin 50}{\sin 32}$$

$$b = \frac{10 \cdot \sin 50}{\sin 32}$$

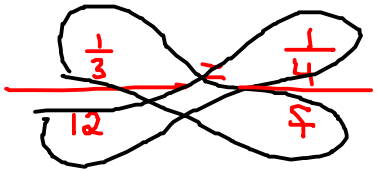
$$b = 14.5$$

2. In $\triangle DEF$, $d = 12$, $\sin D = \frac{1}{3}$, and $\sin F = \frac{1}{4}$. Find f.



$\sin D = \frac{1}{3}$ $\sin F = \frac{1}{4}$

$\frac{\sin D}{d} = \frac{\sin F}{f}$



$\frac{1}{3} \cdot f = 12 \cdot \frac{1}{4}$

$\frac{1}{3} \cdot f = 3$

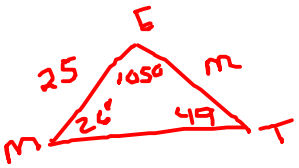
$\frac{1}{3} f = 3$

$\div \frac{1}{3}$ $\div \frac{1}{3}$

$3 \div \frac{1}{3} = 3 \cdot \frac{3}{1} = 9$

$\boxed{f = 9}$

3. In $\triangle MET$, $m\angle M = 26^\circ$, $m\angle E = 105^\circ$, and $t = 25$. Find m.



$26 + 105 = 131$

$180 - 131 = 49$

$\angle T = 49^\circ$

$\frac{\sin M}{m} = \frac{\sin T}{t}$

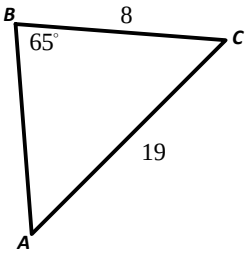
$\frac{\sin 49}{\sin 49} \cdot m = \frac{25 \cdot \sin 26}{\sin 49}$

$m = \frac{25 \cdot \sin 26}{\sin 49}$

$\boxed{m = 14.5}$

4. Solve each triangle.

a)



$\frac{\sin B}{b} = \frac{\sin A}{a}$

$\frac{\sin 65}{19} = \frac{\sin A}{8}$

$\frac{19 \cdot \sin A}{19} = \frac{8 \cdot \sin 65}{19}$

$\sin A = \frac{8 \cdot \sin 65}{19}$

$\sin A = .3816$

$A = 22°$

$22 + 65 = 87$

$180 - 87 = 93$

$\frac{\sin B}{b} = \frac{\sin C}{c}$

$\frac{\sin 65}{19} = \frac{\sin 93}{c}$

$\frac{\sin 65 \cdot c}{\sin 65} = \frac{19 \cdot \sin 93}{\sin 65}$

$c = \frac{19 \cdot \sin 93}{\sin 65}$

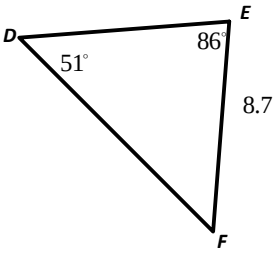
$c = 20.9$

$\angle A = 22°$ $a = 8$

$\angle B = 65°$ $b = 19$

$\angle C = 93°$ $c = 20.9$

b)



$51 + 86 = 137$

$180 - 137 = 43$

$\angle F = 43°$

$\frac{\sin D}{d} = \frac{\sin E}{e}$

$\frac{\sin 51}{8.7} = \frac{\sin 86}{e}$

$\frac{e \cdot \sin 51}{\sin 51} = \frac{8.7 \cdot \sin 86}{\sin 51}$

$e = 11.2$

$\frac{\sin D}{d} = \frac{\sin F}{f}$

$\frac{\sin 51}{8.7} = \frac{\sin 43}{f}$

$\frac{f \cdot \sin 51}{\sin 51} = \frac{8.7 \sin 43}{\sin 51}$

$f = 7.6$

$\angle D = 51°$ $d = 8.7$

$\angle E = 86°$ $e = 11.2$

$\angle F = 43°$ $f = 7.6$