Graphs of Polynomial Functions

Let n be a nonnegative integer and let $a_n,\ a_{n-1}, \ldots a_2,\ a_1$ and a_0 be real numbers with $a_n \neq 0$. The function $f(x) = a_n x^n + a_{n-1}x^{n-1} + \cdots a_2 x^2 + a_1 x^1 + a_0$ is called a polynomial function of x with degree n.

Degree of a Polynomial - the highest exponent in a polynomial function

$f(x) = x^3 - 2x^2 + x + 2$ $\qquad$ $f(x) = -2x^4 + 6x - 1$

Degree = 3 $\qquad$ Degree = 4

Leading Coefficient - the coefficient of the term that contains the polynomial's degree

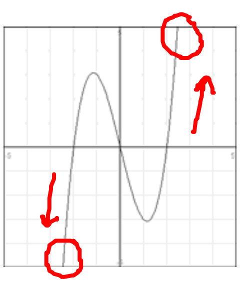
$f(x) = \underline{x^3} - 2x^2 + x + 2$ $\qquad$ $f(x) = \underline{-2x^4} + 6x - 1$

LC = 1 $\qquad$ LC = -2

Leading Coefficient Test

Degree is Odd

Leading Coefficient is Positive

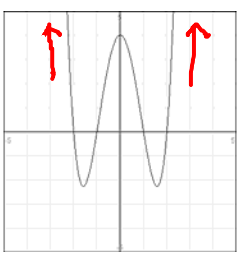


Degree is Odd

Leading Coefficient is Negative

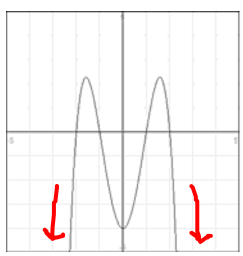
Degree is Even

Leading Coefficient is Positive



Degree is Even

Leading Coefficient is Negative



Zeros of Polynomial Functions

The zero of a polynomial function is the value of x for which $f(x) = 0$.

1. Use the Leading Coefficient Test to determine the right hand and left hand behavior of the graph of the polynomial function.

a) $f(x) = \underbrace{\frac{1}{2}}x^3 + x + 1$

Degree=3 (odd) LC= $\frac{1}{2}$ (pos)

$\boxed{\text{LHB- falls, RHB-rises}}$

b) $f(x) = \underbrace{-3}x^5 - 2x^2 + 1$

Degree=5 (odd) LC=-3 (neg)

$\boxed{\text{LHB- rise, RHB- fall}}$

c) $f(x) = \frac{3x^4 - 2x + 1}{2} = \underbrace{\frac{3}{2}}x^4 - x + \frac{1}{2}$

Degree= 4 (even) LC= $\frac{3}{2}$ (pos)

$\boxed{\text{LHB / RHB - rise}}$

d) $f(x) = -\frac{1}{2}(5x^4 - 2x^3 + x^2 + 5)$

$f(x) = \underbrace{\frac{-5}{2}}x^4 + x^3 - \frac{1}{2}x^2 - \frac{5}{2}$

Degree= 4 (even) LC= $-\frac{5}{2}$ (neg)

$\boxed{\text{LHB/RHB - fall}}$

2. Find all the real zeros of the polynomial function.

Solve / solutions / x-int / roots

a) $f(x) = 16 - x^2$

$16 - x^2 = 0$

$+x^2 \quad +x^2$

$\sqrt{x^2} = \sqrt{16}$

$\boxed{x = \pm 4}$

b) $f(x) = x^2 - 8x + 16$

$x^2 - 8x + 16 = 0$

$(x-4)(x-4) = 0$

$x - 4 = 0 \quad x - 4 = 0$

$\boxed{x = 4} \quad x = 4$

c) $f(x) = 3x^4 + 24x^2 - 60$

$$\frac{3x^4}{3} + \frac{24x^2}{3} - \frac{60}{3} = \frac{0}{3}$$

$$x^4 + 8x^2 - 20 = 0$$

$$(x^2 + 10)(x^2 - 2) = 0$$

$$x^2 + 10 = 0 \qquad x^2 - 2 = 0$$

$$-10 \quad -10 \qquad +2 \quad +2$$

$$\sqrt{x^2} = \sqrt{-10} \qquad \sqrt{x^2} = \sqrt{2}$$

$$\cancel{x = \pm i\sqrt{10}} \qquad \boxed{x = \pm\sqrt{2}}$$

d) $f(x) = x^3 + x^2 - 30x$

$$x^3 + x^2 - 30x = 0$$

$$x(x^2 + x - 30) = 0$$

$$x(x + 6)(x - 5) = 0$$

$$\boxed{x = 0} \qquad x + 6 = 0 \qquad x - 5 = 0$$

$$\boxed{x = -6} \qquad \boxed{x = 5}$$

e) $f(x) = 10x^2 + x - 2$

$$\underline{10x^2} + x - \underline{2} = 0$$

$1 \cdot 10 \qquad 1 \cdot 2$

$2 \cdot 5$

$$(5x - 2)(2x + 1) = 0$$

$4x$

$5x$

$$5x - 2 = 0 \qquad 2x + 1 = 0$$

$$+2 \quad +2 \qquad -1 \quad -1$$

$$\frac{5x}{5} = \frac{2}{5} \qquad \frac{2x}{2} = \frac{-1}{2}$$

$$\boxed{x = 2/5} \qquad \boxed{x = -1/2}$$

3. Find a polynomial function that has the given zeros.

a) $-10,\ 8$

$$x = -10 \qquad x = 8$$

$$x + 10 = 0 \qquad x - 8 = 0$$

$$(x + 10)(x - 8)$$

$$x^2 - 8x + 10x - 80$$

$$\boxed{f(x) = x^2 + 2x - 80}$$

b) $0,\ 2,\ 3$

$$x = 0 \qquad x = 2 \qquad x = 3$$

$$x - 2 = 0 \qquad x - 3 = 0$$

$$x\,(x - 2)(x - 3)$$

$$x\,(x^2 - 3x - 2x + 6)$$

$$x\,(x^2 - 5x + 6)$$

$$\boxed{f(x) = x^3 - 5x^2 + 6x}$$

c) $-4,\ 2,\ -\sqrt{3},\ \sqrt{3}$

$x=-4 \qquad x=2 \qquad x=-\sqrt{3} \qquad x=\sqrt{3}$

$x+4=0 \qquad x-2=0 \qquad x+\sqrt{3}=0 \qquad x-\sqrt{3}=0$

$\underbrace{(x+4)(x-2)}\ \underbrace{(x+\sqrt{3})(x-\sqrt{3})}$

$(x^2-2x+4x-8)\ (x^2-\cancel{\sqrt{3}x}+\cancel{\sqrt{3}x}-3)$

$(x^2+2x-8)\ (x^2-3)$

$x^4-\underline{3x^2}+2x^3-6x-\underline{8x^2}+24$

$$\boxed{f(x)=x^4+2x^3-11x^2-6x+24}$$

d) $1,\ 2-\sqrt{3},\ 2+\sqrt{3}$

$x=1 \qquad x=(2-\sqrt{3}) \qquad x=(2+\sqrt{3})$

$x-1=0 \qquad x-(2-\sqrt{3})=0 \qquad x-(2+\sqrt{3})=0$

$(x-1)\left[x-(2-\sqrt{3})\right]\left[x-(2+\sqrt{3})\right]$

$x^2-x(2+\sqrt{3})-x(2-\sqrt{3})+(2-\sqrt{3})(2+\sqrt{3})$

$\cancel{x^2}-\cancel{2x}-\cancel{\sqrt{3}x}-\cancel{2x}+\cancel{\sqrt{3}x}+4+\cancel{2\sqrt{3}}-\cancel{2\sqrt{3}}-3$

$(x-1)(x^2-4x+1)$

$x^3-4x^2+x-x^2+4x-1$

$$\boxed{f(x)=x^3-5x^2+5x-1}$$