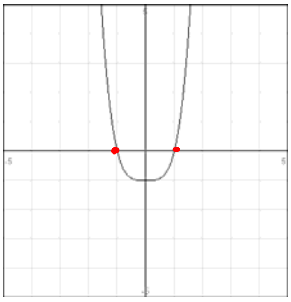
Complex Zeros of Polynomial Functions

1. Find all the zeros of the polynomial function.

a) $f(x) = x^4 - 1$

Degree = 4



$x^4 - 1 = 0$

$(x^2+1)(x^2-1) = 0$

$x^2 + 1 = 0$ $\quad$ $x^2 - 1 = 0$

$-1 \quad -1$ $\quad$ $+1 \quad +1$

$\sqrt{x^2} = \sqrt{-1}$ $\quad$ $\sqrt{x^2} = \sqrt{1}$

$x = \pm i$ $\quad$ $x = \pm 1$

$\boxed{\text{Zeros: } \pm 1, \pm i}$

b) $f(x) = \underbrace{1}_{\frac{q}{1 \cdot 1}}x^3 - 3x^2 + 4x \underbrace{-2}_{\frac{p}{1 \cdot 2}}$

p: $\pm 1, \pm 2$

q: ± 1

p/q: $\pm 1, \pm 2$

$$\begin{array}{r|rrrr} * \; 1 & 1 & -3 & 4 & -2 \\ & & 1 & -2 & 2 \\ \hline & 1x^2 & -2x & 2 & \boxed{0} \end{array}$$

$x^2 - 2x + 2 = 0$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$a = 1 \quad b = -2 \quad c = 2$

$x = \dfrac{-(-2) \pm \sqrt{(-2)^2 - 4(1)(2)}}{2(1)}$

$x = \dfrac{2 \pm \sqrt{-4}}{2}$

$x = \dfrac{\cancel{2}^{1} \pm \cancel{2}^{1} i}{\cancel{2}_{1}}$

$x = 1 \pm i$

$\boxed{\text{Zeros: } 1, 1 \pm i}$

c) $f(x) = \underbrace{1}_{q}x^4 + 6x^3 + 10x^2 + 6x + \underbrace{9}_{p}$

q: $1 \cdot 1$

p: $1 \cdot 9$, $3 \cdot 3$

p: $\pm 1, \pm 3, \pm 9$

q: ± 1

p/q: $\pm 1, \pm 3, \pm 9$

-1	1	6	10	6	9
		-1	-5	-5	-1
	1	5	5	1	$\boxed{8}$

* -3	1	6	10	6	9
		-3	-9	-3	-9
	$1x^3$	$3x^2$	$1x$	3	$\boxed{0}$

$x^3 + 3x^2 + x + 3 = 0$

$x^2(x+3) + 1(x+3) = 0$

GCF $= x^2$ GCF $= 1$

$(x+3)(x^2+1) = 0$

$x + 3 = 0$ $x^2 + 1 = 0$

$-3 \quad -3$ $-1 \quad -1$

$x = -3$ $\sqrt{x^2} = \sqrt{-1}$

$x = \pm i$

$\boxed{\text{zeros: } -3, -3, \pm i}$

2. Find a polynomial function with integer coefficients that has the given zeros.

a) $\frac{1}{2}$, $\pm 3i$

$x = \frac{1}{2}$ $x = 3i$ $x = -3i$

$x - \frac{1}{2} = 0$ $x - 3i = 0$ $x + 3i = 0$

$(x - 1/2)(\overbrace{x - 3i}^{F})(\underbrace{x + 3i}_{L})$

$(x - 1/2)(x^2 - 9i^2)$

$(x - 1/2)(x^2 + 9)$

$(2x - 1)(x^2 + 9)$

$2x^3 + 18x - x^2 - 9$

$\boxed{f(x) = 2x^3 - x^2 + 18x - 9}$

b) $0,\ 0,\ -2 \pm 4i$

$x = 0 \qquad x = 0 \qquad x = (-2+4i) \qquad x = (-2-4i)$

$x - (-2+4i) = 0 \qquad x - (-2-4i) = 0$

$$(x)(x)\left[x - (-2+4i)\right]\left[x - (-2-4i)\right]$$

$$x^2\left[x^2 - x(-2-4i) - x(-2+4i) + (-2+4i)(-2-4i)\right]$$

$$x^2\left[x^2 + 2x + \cancel{4xi} + 2x - \cancel{4xi} + 4 - 16i^2\right]$$

$$x^2\left[x^2 + 4x + 20\right]$$

$$\boxed{f(x) = x^4 + 4x^3 + 20x^2}$$

3. Use the given zeros to find all the zeros of the function.

a) $f(x) = 2x^4 - x^3 + 7x^2 - 4x - 4$

Zero $= 2i$, $-2i$

$2i$	2	-1	7	-4	-4
		$4i$	$-8-2i$	$-2i+4$	4
	$2x^3$	$4i-1\,x^2$	$-1-2i\,x$	$-2i$	0

$2i(4i-1) = 8i^2 - 2i = -8-2i$

$2i(-1-2i) = -2i - 4i^2 = -2i+4$

$2i(-2i) = -4i^2 = 4$

$-2i$	2	$4i-1$	$-1-2i$	$-2i$
		$-4i$	$2i$	$2i$
	$2x^2$	$-1\,x$	-1	0

$2x^2 - x - 1 = 0$

$(2x+1)(x-1) = 0$

$2x+1=0$ $\quad$ $x-1=0$

$-1 \quad -1$ $\quad$ $+1 \quad +1$

$2x = -1$ $\quad$ $x = 1$

$x = -1/2$

$\boxed{\text{zeros: } \pm 2i, -1/2, 1}$

b) $f(x) = x^3 - 3x^2 - 15x + 125$

Zero $= 4+3i$, $4-3i$

$4+3i$	1	-3	-15	125
		$4+3i$	$-5+15i$	-125
	$1x^2$	$1+3i\,x$	$-20+15i$	0

$(4+3i)(1+3i) = 4+12i+3i+9i^2 = 4+15i-9 = -5+15i$

$(4+3i)(-20+15i) = -80+\cancel{60i}-\cancel{60i}+45i^2 = -80-45 = -125$

$4-3i$	1	$1+3i$	$-20+15i$
		$4-3i$	$20-15i$
	$1x$	5	0

$5(4-3i) = 20-15i$

$\boxed{\text{zeros: } -5, 4 \pm 3i}$

$x+5=0$

$x=-5$