Compound Interest and Radioactive Decay

Compound Interest

$A = P\left(1+\frac{r}{n}\right)^{nt}$ Use when given *n* compoundings per year

$A = Pe^{rt}$ Use when compounded continuously

A = balance (future amount)
P = principal (original amount)
r = interest rate in decimal form
n = number of compoundings per year
t = time in years

A = balance (future amount)
P = principal (original amount)
e = the natural base
r = interest rate in decimal form
t = time in years

Values for *n*
Annually: *n* = 1
Biannually/Semiannually: *n* = 2
Quarterly: *n* = 4
Monthly: *n* = 12
Weekly: *n* = 52
Daily: *n* = 365

1. A total of $11,000 is invested at an annual interest rate of 7.5%, compounded monthly. Find the balance in the account after 5 years.

$A = P\left(1+\frac{R}{N}\right)^{Nt}$

$P = 11{,}000$
$R = .075$
$N = 12$
$t = 5$
$A = ?$

$A = 11{,}000\left(1+\frac{.075}{12}\right)^{(12)(5)}$
$A = 11{,}000(1.00625)^{60}$
$A = 15{,}986.24$

$\boxed{\text{Balance} = \$15{,}986.24}$

2. The balance after 3 years in an account is $2,500. How much was originally invested if the annual interest rate is 7.25%, compounded continuously?

$A = Pe^{Rt}$

$A = 2500$
$t = 3$
$R = .0725$
$P = ?$

$2500 = Pe^{(.0725)(3)}$
$\frac{2500}{e^{.2175}} = \frac{P\cancel{e^{.2175}}}{\cancel{e^{.2175}}}$

$P = 2011.32$

$\boxed{\text{original } \$2011.32}$

3. Katie deposits $5,000 into an account that pays 6.25% interest, compounded continuously. How long will it take for the money to triple?

$A = Pe^{Rt}$

$P = 5000$

$R = .0625$

$A = 15{,}000$

$t = ?$

$\frac{15000}{5000} = \frac{\cancel{5000}e^{.0625t}}{\cancel{5000}}$

$3 = e^{.0625t}$

$\ln 3 = \ln e^{.0625t}$

$\ln 3 = .0625t \cancel{\ln e}$

1

$\frac{\ln 3}{.0625} = \frac{\cancel{.0625}t}{\cancel{.0625}}$

$\boxed{t = 18 \text{ years}}$

<u>Radioactive Decay</u>

$$\frac{1}{2} = e^{kt}$$

Use to find the value of k when given the half-life in t years.

$$y = ae^{kt}$$

y = amount after t years
a = initial amount
e = the natural base
k = constant
t = time in years

4. The half-life of Radium 226 is 1,620 years. Determine the amount of Radium 226 after 1,000 years if the initial amount was 25 grams.

$\frac{1}{2} = e^{Kt}$ $t = 1{,}620$

$\frac{1}{2} = e^{1620K}$

$\ln \frac{1}{2} = \ln e^{1620K}$

$\frac{\ln \frac{1}{2}}{1620} = \frac{1620K \cancel{\ln e}}{1620}$ 1

$K = -.000428$

$y = ae^{Kt}$

$K = -.000428$

$t = 1000$

$A = 25$

$y = ?$

$y = 25e^{(-.000428)(1000)}$

$y = 25e^{-.428}$

$\boxed{y = 16.295 \text{ g}}$

5. The half-life of Carbon 14 is 5,730 years. What percent of the present amount of Carbon 14 will remain after 1000 years?

$\frac{1}{2} = e^{kt}$ $\quad t = 5730$

$\frac{1}{2} = e^{5730k}$

$\ln \frac{1}{2} = \ln e^{5730k}$

$\frac{\ln \frac{1}{2}}{5730} = \frac{5730k \ln \cancel{e}}{5730}$

$k = -.000121$

$y = ae^{kt}$

$k = -.000121$

$t = 1000$

$\frac{y}{a} = ?$

$\frac{y}{a} = \frac{ae^{(-.000121)(1000)}}{a}$

$\frac{y}{a} = e^{(-.000121)(1000)}$

$\frac{y}{a} = e^{-.121}$

$\frac{y}{a} = .89$ $\boxed{89\%}$