Mathematical Induction

Steps to Prove by Mathematical Induction

Step 1: Find a_1 and S_1. N = 1

Step 2: Find a_k, S_k, a_{k+1} and S_{k+1}. N = K N = K+1

Step 3: Show $\underbrace{S_{k+1} = a_{k+1} + S_k}$.

$$a_1 + a_2 + a_3 + \cdots + a_n = S_n$$

Directions: Use Mathematical Induction to prove each formula.

1. $\underset{a_1}{1} + \underset{a_2}{4} + \underset{a_3}{7} + \underset{a_4}{10} + \cdots + \underset{a_n}{(3n-2)} = \underset{S_n}{\frac{n}{2}(3n-1)}$

Step1: N = 1

$3(1) - 2 = \frac{1}{2}(3 \cdot 1 - 1)$

$1 = \frac{1}{2}(2)$

$1 = 1$ ✓

Step 2: N = K

$a_K = 3K - 2$

$S_K = \frac{K}{2}(3K-1)$

N = K+1

$a_{K+1} = 3(K+1) - 2$

$= 3K + 3 - 2$

$= 3K+1$

$S_{K+1} = \frac{K+1}{2}(3(K+1) - 1)$

$= \frac{K+1}{2}(3K+3-1)$

$= \frac{K+1}{2}(3K+2)$

Step3: $S_{K+1} = a_{K+1} + S_K$

$\frac{K+1}{2}(3K+2) = \frac{3K+1}{1} \cdot \frac{2}{2} + \frac{K(3K-1)}{2}$

LCD = 2

$\frac{K+1}{2}(3K+2) = \frac{2(3K+1) + K(3K-1)}{2}$

$= \frac{6K + 2 + 3K^2 - K}{2} = \frac{3K^2 + 5K + 2}{2}$

$\frac{(K+1)(3K+2)}{2} = \frac{(3K+2)(K+1)}{2}$ ✓

2. $2\left[1+3+3^2+3^3+\cdots+3^{n-1}\right]=3^n-1$

a_N S_N

$a_1 = 2\cdot 1 = 2$

$a_2 = 2\cdot 3 = 6$

Step 1: $N=1$

$a_N = S_N$

$a_1 = 2\cdot 3^{1-1}$

$a_1 = 2\cdot 3^0$

$a_1 = 2$

$2 = 2$ ✓

$S_1 = 3^1 - 1$

$S_1 = 2$

Step 2: $N=K$

$a_K = 2\cdot 3^{K-1}$

$S_K = 3^K - 1$

$N = K+1$

$a_{K+1} = 2\cdot 3^{K+1-1}$

$a_{K+1} = 2\cdot 3^K$

$S_{K+1} = 3^{K+1} - 1$

Step 3: $S_{K+1} = a_{K+1} + S_K$

$3^{K+1} - 1 = \underbrace{2\cdot 3^K + 1\cdot 3^K}_{} - 1$

$= 3^1\cdot 3^K - 1$

$3^{K+1} - 1 = 3^{K+1} - 1$ ✓

3. $\sum_{i=1}^{n} i(i+1) = \frac{n(n+1)(n+2)}{3}$

$2 + 6 + 12 + \cdots N(N+1) = \frac{N(N+1)(N+2)}{3}$

a_N $\qquad S_N$

$i=1 \quad 1(1+1)=2$

$i=2 \quad 2(2+1)=6$

$i=3 \quad 3(3+1)=12$

Step 1: $N=1$

$a_1 = 1(1+1) = 2$

$S_1 = \frac{1(1+1)(1+2)}{3}$

$= \frac{1(2)(3)}{3}$

$= 2$

$a_1 = S_1$

$2 = 2$ ✓

Step 2: $N=K$ $\quad a_K = K(K+1)$ $\qquad N=K+1$

$S_K = \frac{K(K+1)(K+2)}{3}$

$a_{K+1} = (K+1)(K+1+1)$

$= (K+1)(K+2)$

$S_{K+1} = \frac{(K+1)(K+1+1)(K+1+2)}{3}$

$S_{K+1} = \frac{(K+1)(K+2)(K+3)}{3}$

Step 3: $S_{K+1} = a_{K+1} + S_K$

$\frac{(K+1)(K+2)(K+3)}{3} = \frac{3 \cdot (K+1)(K+2)}{3 \cdot 1} + \frac{K(K+1)(K+2)}{3}$

LCD = 3

$= \frac{3(K+1)(K+2) + K(K+1)(K+2)}{3}$

GCF = $(K+1)(K+2)$

$\frac{(K+1)(K+2)(K+3)}{3} = \frac{(K+1)(K+2)(3+K)}{3}$ ✓