

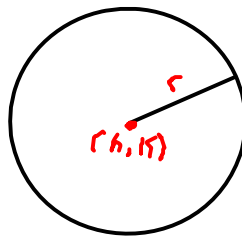
Conic Sections - Circles

Standard Form for the Equation of a Circle

$$(x-h)^2 + (y-k)^2 = r^2$$

Center = (h, k)

Radius = r



General Form for the Equation of a Circle

$$x^2 + y^2 + Ax + By + C = 0$$

A , B and C are constants.

x^2 and y^2 have the same coefficients.

1. Find the center and radius for each circle.

a) $(x+1)^2 + (y-2)^2 = 49$

center $(-1, 2)$

Radius = $\sqrt{49} = 7$

b) $x^2 + \left(y - \frac{1}{2}\right)^2 = 1$

center $(0, \frac{1}{2})$

Radius = $\sqrt{1} = 1$

c) $x^2 + y^2 = 5$

center $(0, 0)$

Radius = $\sqrt{5}$

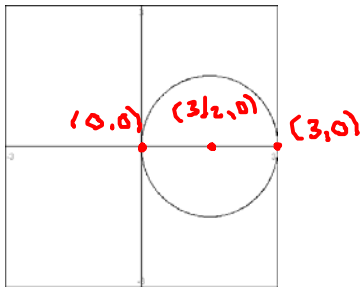
$$d) \left(x + \frac{1}{2}\right)^2 + \left(y - \frac{3}{2}\right)^2 = \frac{9}{4}$$

Center $\boxed{\left(-\frac{1}{2}, \frac{3}{2}\right)}$

$$\text{Radius} = \sqrt{\frac{9}{4}} = \frac{\sqrt{9}}{\sqrt{4}} = \boxed{\frac{3}{2}}$$

2. Find an equation for the circle.

a)



$$\begin{aligned} \text{midpoint} &= \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right) \\ &= \left(\frac{0 + 3}{2}, \frac{0 + 0}{2}\right) = \left(\frac{3}{2}, 0\right) \end{aligned}$$

$$\text{center } \left(\frac{3}{2}, 0\right) \quad \text{Radius} = \frac{3}{2}$$

$$\begin{aligned} (x-h)^2 + (y-k)^2 &= R^2 \\ (x - \frac{3}{2})^2 + y^2 &= \left(\frac{3}{2}\right)^2 \end{aligned}$$

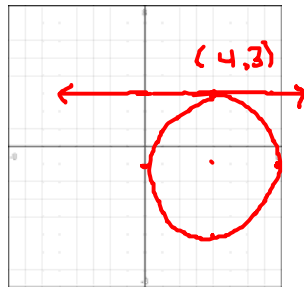
$$\boxed{(x - \frac{3}{2})^2 + y^2 = \frac{9}{4}}$$

b) Center is at $(4, -1)$ and tangent to the line $y = 3$.

$$R = 4$$

$$(x-h)^2 + (y-k)^2 = R^2$$

$$\boxed{(x-4)^2 + (y+1)^2 = 16}$$



c) Center is at $(-1, -3)$ and tangent to the line $3x + 4y = 10$.

$$\begin{aligned} 3x + 4y &= 10 \\ -3x & \quad -3x \\ \hline 4y &= -3x + 10 \\ \frac{4y}{4} &= \frac{-3x + 10}{4} \\ y &= -\frac{3}{4}x + \frac{5}{2} \end{aligned}$$

$$m_a = \frac{4}{3}$$

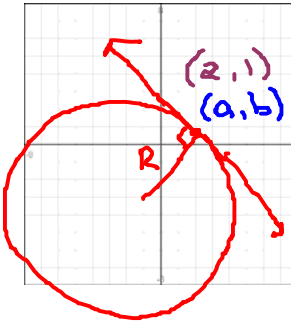
$$(a, b) (-1, -3)$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = -\frac{3}{4}$$

$$b = (0, \frac{5}{2})$$

$$\begin{aligned} 4 &= -3 - b \\ 3 &= -1 - a \end{aligned}$$



$$3x + 4y = 10$$

$$* 3a + 4b = 10$$

$$\begin{aligned} 4(-1 - a) &= 3(-3 - b) \\ -4 - 4a &= -9 - 3b \\ +9 + 4a &+9 + 4a \\ * 5 &= 4a - 3b \end{aligned}$$

$$(-1, -3) (2, 1)$$

$$\begin{aligned} 3(3a + 4b = 10) & \quad 9a + 12b = 30 & 3(2) + 4b = 10 \\ 4(4a - 3b = 5) & \quad 16a - 12b = 20 & 6 + 4b = 10 \end{aligned}$$

$$\frac{25a}{25} = \frac{50}{25}$$

$$a = 2$$

$$\frac{4b}{4} = \frac{4}{4}$$

$$b = 1$$

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$d = \sqrt{(2 - (-1))^2 + (-1 - (-3))^2}$$

$$d = \sqrt{3^2 + 2^2}$$

$$d = \sqrt{9 + 4}$$

$$d = \sqrt{13}$$

$$d = 5$$

$$C: (-1, -3)$$

$$R = 5$$

$$\begin{aligned} (x - h)^2 + (y - k)^2 &= R^2 \\ (x + 1)^2 + (y + 3)^2 &= 25 \end{aligned}$$

3. Find the standard equation for the circle and sketch the graph.

$$\frac{2x^2}{2} + \frac{2y^2}{2} - \frac{8x}{2} + \frac{12y}{2} + \frac{2}{2} = 0$$

$$x^2 + y^2 - 4x + 6y + 1 = 0$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = -1 + 4 + 9$$

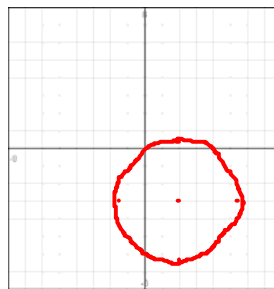
$$\frac{4}{2} = (2)^2 = 4 \quad \frac{9}{2} = (3)^2 = 9$$

$$(x^2 - 4x + 4) + (y^2 + 6y + 9) = 12$$

$$(x - 2)^2 + (y + 3)^2 = 12$$

$$\text{center: } (2, -3)$$

$$R = \sqrt{12} \approx 3.5$$



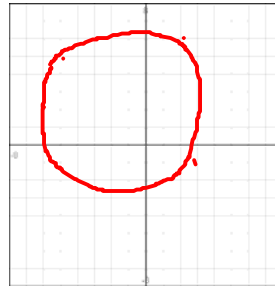
4. Find the standard equation for the circle that contains the three points $(2,6)$, $(3,-1)$ and $(-5,5)$.

$$x^2 + y^2 + Ax + By + C = 0$$

$$\begin{aligned} (2,6) \quad 2^2 + 6^2 + 2A + 6B + C &= 0 \\ 4 + 36 + 2A + 6B + C &= 0 \\ 2A + 6B + C &= -40 \quad (1) * \end{aligned}$$

$$\begin{aligned} (3,-1) \quad 3^2 + (-1)^2 + 3A - 1B + C &= 0 \\ 9 + 1 + 3A - 1B + C &= 0 \\ 3A - B + C &= -10 \quad (2) * \end{aligned}$$

$$\begin{aligned} (-5,5) \quad (-5)^2 + (5)^2 - 5A + 5B + C &= 0 \\ 25 + 25 - 5A + 5B + C &= 0 \\ -5A + 5B + C &= -50 \quad (3) * \end{aligned}$$



$$\begin{aligned} 2A + 6B + C &= -40 \\ 3A - B + C &= -10 \\ -5A + 5B + C &= -50 \end{aligned}$$

Elim. C using (1) + (2)

$$\begin{aligned} 2A + 6B + C &= -40 \\ -3A + B - C &= 10 \\ \hline -A + 7B &= -30 \quad (4) * \end{aligned}$$

Elim. C using (2) + (3)

$$\begin{aligned} 3A - B + C &= -10 \\ 5A - 5B - C &= 50 \\ \hline 8A - 6B &= 40 \quad (5) * \end{aligned}$$

use (4) + (5) to find A + B

$$\begin{aligned} 8(-A + 7B = -30) &\rightarrow -8A + 56B = -240 \\ 8A - 6B = 40 & \\ \hline 50B &= -200 \\ \frac{50B}{50} &= \frac{-200}{50} \\ B &= -4 \end{aligned}$$

$$\begin{aligned} \text{use (4) to find A} \\ -A + 7(-4) &= -30 \\ -A - 28 &= -30 \\ +28 \quad +28 & \\ -A &= -2 \\ A &= 2 \end{aligned}$$

$$\begin{aligned} \text{use (1) to} \\ \text{find C} \\ 2A + 6B + C &= -40 \\ 2(2) + 6(-4) + C &= -40 \\ 4 - 24 + C &= -40 \\ -20 + C &= -40 \\ +20 \quad +20 & \\ C &= -20 \end{aligned}$$

$$\begin{aligned} x^2 + y^2 + Ax + By + C &= 0 \\ x^2 + y^2 + 2x - 4y - 20 &= 0 \\ \underbrace{\hspace{10em}}_{+20 \quad +20} & \end{aligned}$$

$$\begin{aligned} (x^2 + 2x + 1) + (y^2 - 4y + 4) &= 20 \\ \frac{2}{2} = (1)^2 = 1 \quad \frac{4}{2} = (2)^2 = 4 & \quad +1 \\ & \quad +4 \end{aligned}$$

$$\boxed{(x+1)^2 + (y-2)^2 = 25}$$