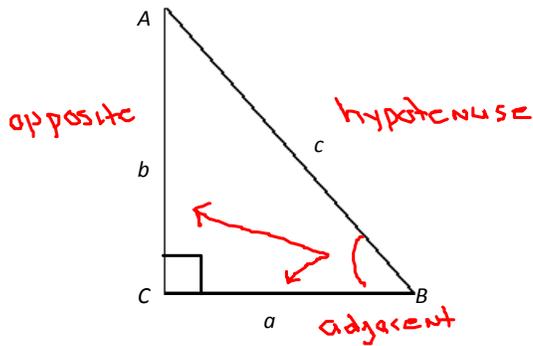


Trigonometric Functions of an Angle - Part 2



Definitions of Trigonometric Functions

"sine" $\sin \angle = \frac{\text{opposite}}{\text{hypotenuse}}$ $\sin B = \frac{b}{c}$

"cosecant" $\csc \angle = \frac{\text{hypotenuse}}{\text{opposite}}$ $\csc B = \frac{c}{b}$

"cosine" $\cos \angle = \frac{\text{adjacent}}{\text{hypotenuse}}$ $\cos B = \frac{a}{c}$

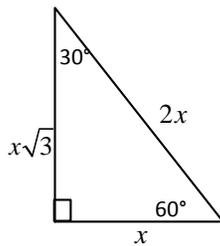
"secant" $\sec \angle = \frac{\text{hypotenuse}}{\text{adjacent}}$ $\sec B = \frac{c}{a}$

"tangent" $\tan \angle = \frac{\text{opposite}}{\text{adjacent}}$ $\tan B = \frac{b}{a}$

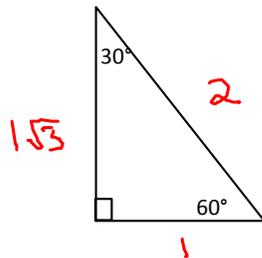
"cotangent" $\cot \angle = \frac{\text{adjacent}}{\text{opposite}}$ $\cot B = \frac{a}{b}$

Special Right Triangles

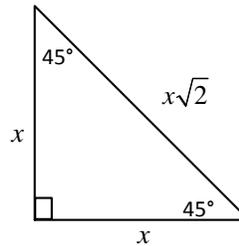
30° - 60° - 90°



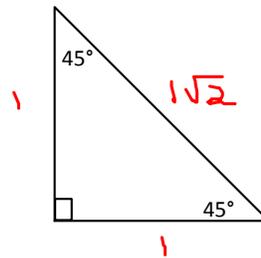
$x=1$



45° - 45° - 90°



$x=1$

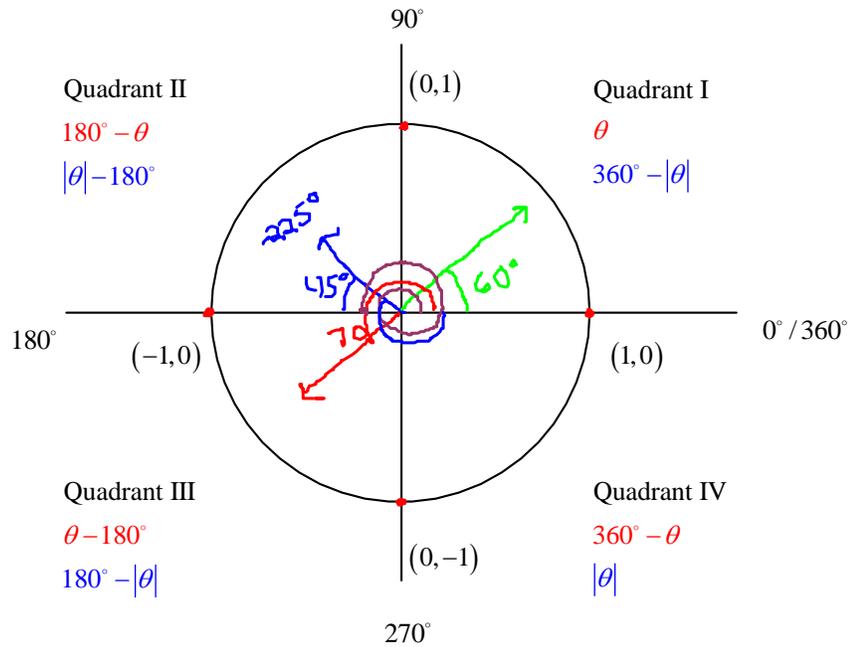


Reference Angles

Positive Angles

Negative Angles

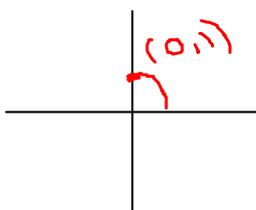
Angle (θ)	Reference Angle
250°	$250 - 180 = 70$
-225°	$ -225 - 180 = 45$
60°	60
540°	NONE



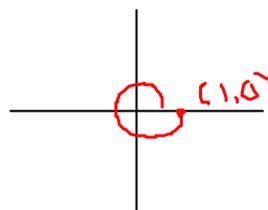
1. Evaluate the trigonometric function of each quadrant angle.

a) $\cos \frac{\pi}{2} = \frac{180}{2} = 90^\circ$

b) $\tan 2\pi = \frac{360}{\pi} = 360^\circ$



$\cos 90 = 0$



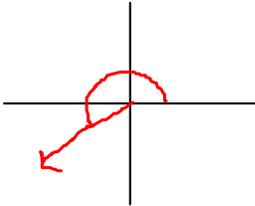
$\sin 360 = 0 = \boxed{0}$

$$\boxed{\cos 10 - 1}$$

$$+\cos 360 - \frac{\quad}{\cos 360} \quad \boxed{\quad}$$

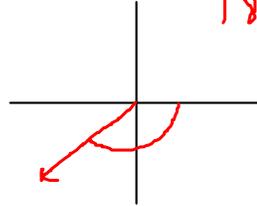
2. Find the reference angle for each angle θ in standard position.

a) $\theta = 200^\circ$



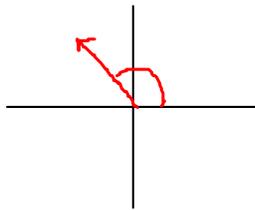
$$\begin{aligned} \theta &= 180 \\ 200 - 180 &= \boxed{20} \end{aligned}$$

b) $\theta = -130^\circ$



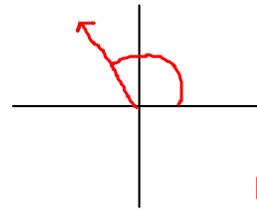
$$\begin{aligned} 180 - |0| \\ 180 - |-130| \\ \boxed{50} \end{aligned}$$

c) $\theta = \frac{5\pi}{9} \cdot \frac{180}{\pi} = 100^\circ$



$$\begin{aligned} 180 - 0 \\ 180 - 100 \\ \boxed{80} \end{aligned}$$

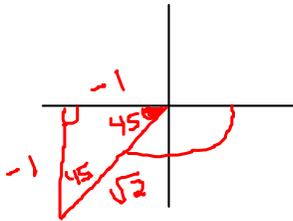
d) $\theta = 1.8 \cdot \frac{180}{\pi} = 1.8 \times \frac{180}{3.14}$



$$\begin{aligned} 103^\circ \\ 180 - 0 \\ 180 - 103 \\ \boxed{77} \end{aligned}$$

3. Evaluate the six trigonometric functions of angle θ .

a) $\theta = -135^\circ$



$$180 - 101$$

$$180 - |-135|$$

$$\text{Reference } \angle = 45^\circ$$

$$\sin -135^\circ = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

$$\cos -135^\circ = -\frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = -\frac{\sqrt{2}}{2}$$

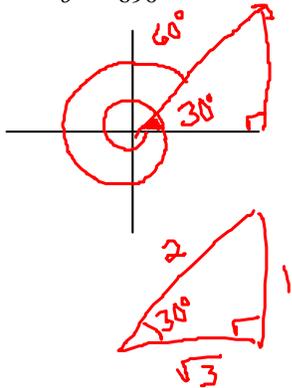
$$\tan -135^\circ = \frac{-1}{-1} = 1$$

$$\csc -135^\circ = -\sqrt{2}$$

$$\sec -135^\circ = -\sqrt{2}$$

$$\cot -135^\circ = 1$$

b) $\theta = -690^\circ$



$$\sin -690^\circ = \frac{1}{2}$$

$$\cos -690^\circ = \frac{\sqrt{3}}{2}$$

$$\tan -690^\circ = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

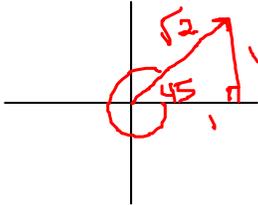
$$\csc -690^\circ = 2$$

$$\sec -690^\circ = \frac{2}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{2\sqrt{3}}{3}$$

$$\cot -690^\circ = \frac{\sqrt{3}}{1} = \sqrt{3}$$

c) $\theta = -\frac{7\pi}{4} \cdot \frac{45}{\pi} = -315^\circ$

$$360 - 101$$



$$360 - 1 - 315$$

$$\text{Reference } \angle = 45^\circ$$

$$\sin \frac{-7\pi}{4} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\cos \frac{-7\pi}{4} = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\tan \frac{-7\pi}{4} = \frac{1}{1} = 1$$

$$\csc \frac{7\pi}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\sec \frac{7\pi}{4} = \frac{\sqrt{2}}{1} = \sqrt{2}$$

$$\cot \frac{-7\pi}{4} = \frac{1}{1} = 1$$

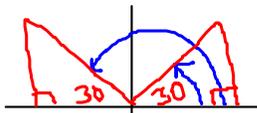
4. Find two solutions of the equation for θ in degrees ($0^\circ \leq \theta \leq 360^\circ$) and radians ($0 \leq \theta \leq 2\pi$).

a) $\sin \theta = \frac{1}{2}$

I II

b) $\tan \theta = -1$

II IV

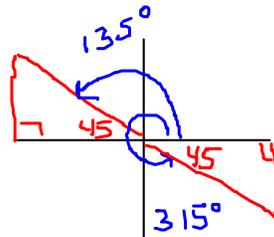


$$30^\circ, 150^\circ$$

$$\frac{\pi}{6}, \frac{5\pi}{6}$$

$$30^\circ \cdot \frac{\pi}{180} = \frac{\pi}{6}$$

$$150^\circ \cdot \frac{\pi}{180} = \frac{5\pi}{6}$$



$$135^\circ, 315^\circ$$

$$\frac{3\pi}{4}, \frac{7\pi}{4}$$

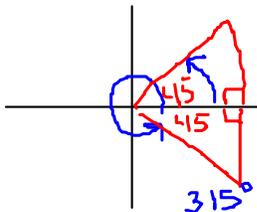
$$135^\circ \cdot \frac{\pi}{180} = \frac{3\pi}{4}$$

$$315^\circ \cdot \frac{\pi}{180} = \frac{7\pi}{4}$$

c) $\sec \theta = \sqrt{2}$

$$\cos \theta = \frac{1}{\sqrt{2}}$$

I IV



$$45^\circ \cdot \frac{\pi}{180} = \frac{\pi}{4}$$

$$315^\circ \cdot \frac{\pi}{180} = \frac{7\pi}{4}$$

$$45^\circ, 315^\circ$$

$$\frac{\pi}{4}, \frac{7\pi}{4}$$

| 315°

$$\overset{7}{315^\circ} \cdot \frac{\overset{100}{\pi}}{180} = \frac{7\pi}{4}$$

$$\boxed{\pi/4, 7/4}$$