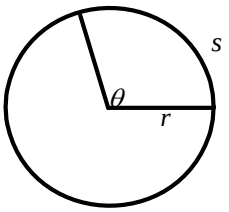
Arc Length, Area of a Sector, Linear Speed and Angular Speed

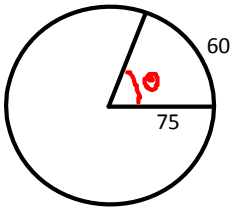
Arc Length



$s = r\theta$

s: arc length
r: radius
θ: central angle measured in radians

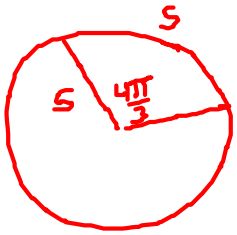
1. Find the measure of the angle in radians.



R=75
S=60

S=RΘ
60/75 = 75Θ/75
Θ=.8 Radians

2. Find the length of the arc on a circle with a radius of 5 meters intercepted by a central angle of $\frac{4\pi}{3}$ radians.



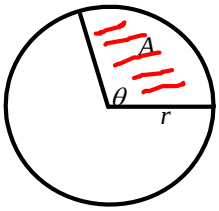
R= 5
Θ= 4π/3

S=RΘ

S= 5 . 4π/3

S= 5x 4x3.14 / 3 = 20.9 meters

Area of a Sector



$$A = \frac{1}{2}\theta r^2$$

A: area of the sector
r: radius
θ: central angle measured in radians

3. Find the area of the sector of a circle with a central angle of 3.4 radians and a radius of 5.2 inches.

$A = \frac{1}{2}\theta R^2$

$A = \frac{1}{2}(3.4)(5.2)^2$

$A = 45.97 \text{ in}^2$

4. A sprinkler can water a lawn up to a distance of 50 feet. It turns through an angle of 120°. Find the area of the lawn that can be watered by the sprinkler.

$R = 50$

$\theta = 120° \cdot \frac{\pi}{180} = \frac{120(3.14)}{180} = 2.09$

$A = \frac{1}{2}\theta R^2$

$A = \frac{1}{2}(2.09)(50)^2$

$A = 2617.99 \text{ feet}^2$

Linear Speed - Measures how fast a particle is moving at a constant speed along a circular arc.

$$\text{Linear Speed} = \frac{r\theta}{t}$$

r: radius
θ: central angle in radians
t: time

5. The second hand of a clock is 5.6 inches long. Find the linear speed of the tip of the second hand.

$R = 5.6$ in

$\theta = 2\pi$

$t = 60$ sec

$LS = \frac{R\theta}{t}$

$LS = \frac{(5.6)(2\pi)}{60} = .59$ in/sec

6. A helicopter blade is 3 meters long and rotates at 430 revolutions per minute. What is the linear velocity of the tip of the blade?

$LS = \frac{R\theta}{t}$

$R = 3$ m

$\theta = 2\pi \times 430$

$t = 1$ min

$LS = \frac{(3)(2\pi)(430)}{1}$

$LS = 8105.31$ m/min

Angular Speed - Measures how fast the central angle is changing as a particle is moving at a constant speed along a circular arc.

$$\text{Angular Speed} = \frac{\theta}{t} \text{ or } \frac{s}{rt}$$

s: arc length

r: radius

θ: central angle in radians

t: time

7. Through what angle does the drive shaft of a car rotate in one second when the tachometer reads 2,500 revolutions per minute?

$\theta = 2\pi \times 2500$

$t = 1$ min/60 sec

$AS = \frac{\theta}{t}$

$AS = \frac{2\pi \times 2500}{60} = 261.80$ radians

8. A car is moving at a rate of 55 miles per hour and the diameter of each wheel is 2.42 feet.

a) What is the rotational speed of the wheels in revolutions per minute?

$D = 2.42$

$R = 1.21$

$$\frac{55 \text{ mi}}{1 \text{ hr}} \times \frac{5280 \text{ ft}}{1 \text{ mi}} \times \frac{1 \text{ hr}}{60 \text{ min}} = 4840 \text{ ft/min}$$

$$\frac{4840}{1.21(2\pi)} = \boxed{636.62 \text{ Rev/min}}$$

b) What is the angular speed of the wheels in radians per minute?

$$636.62 \times 2\pi = \boxed{4000 \text{ rad/min}}$$