

Inverse Trigonometric Functions

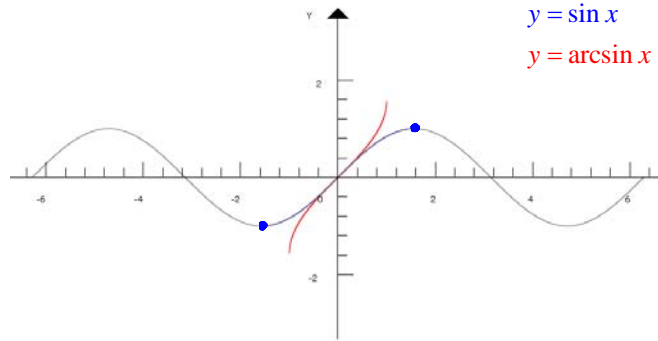
$$y = \arcsin x$$

Positive - Quadrant I

Negative - Quadrant IV

Domain: $[-1, 1]$

Range: $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$



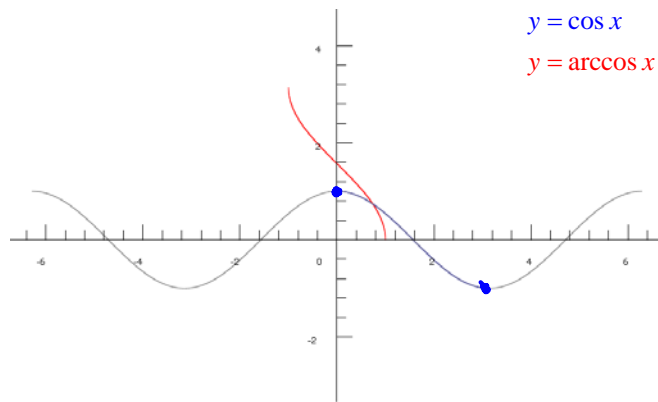
$$y = \arccos x$$

Positive - Quadrant I

Negative - Quadrant II

Domain: $[-1, 1]$

Range: $[0, \pi]$



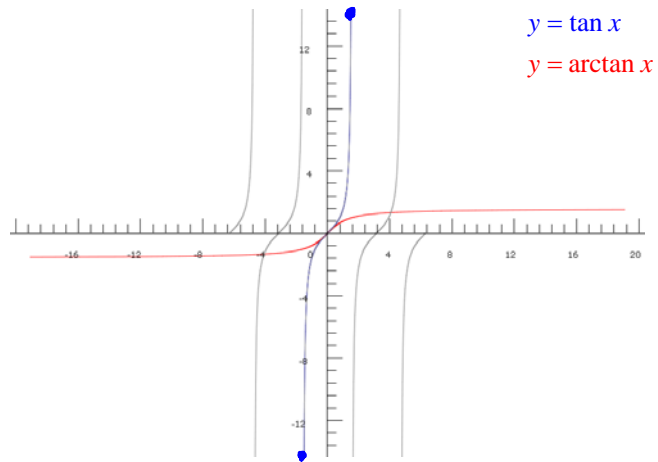
$$y = \arctan x$$

Positive - Quadrant I

Negative - Quadrant IV

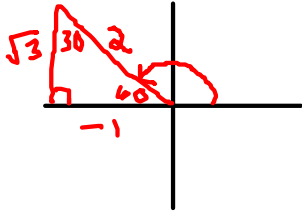
Domain: $[-\infty, \infty]$

Range: $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

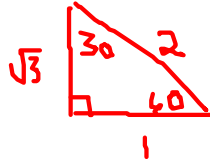


Directions: Find the exact value of each in radians.

1. $\arccos\left(-\frac{1}{2}\right)$

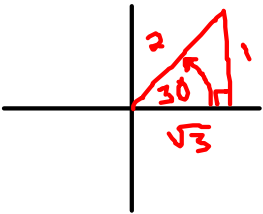


Positive I
Negative II

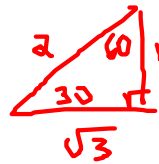


$$120^\circ \cdot \frac{\pi}{180} = \boxed{\frac{2\pi}{3}}$$

2. $\arctan\left(\frac{\sqrt{3}}{3}\right)$



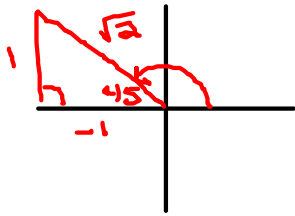
Positive I
Negative III



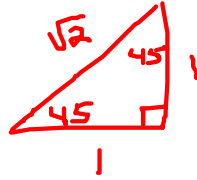
$$\tan 30 = \frac{1}{\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}} = \frac{\sqrt{3}}{3}$$

$$30^\circ \times \frac{\pi}{180} = \boxed{\frac{\pi}{6}}$$

3. $\arccos\left(-\frac{\sqrt{2}}{2}\right)$



Positive I
Negative II



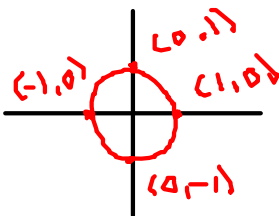
$$\cos 45 = \frac{1}{\sqrt{2}} \cdot \frac{\sqrt{2}}{\sqrt{2}} = \frac{\sqrt{2}}{2}$$

$$\frac{3}{4} \pi \cdot \frac{180}{180} = \boxed{\frac{3\pi}{4}}$$

4. $\sin^{-1}3$

No solution

5. $\arcsin(0)$

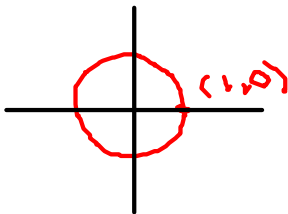


$$0^\circ, 180^\circ$$

$$0^\circ = \boxed{0 \text{ radians}}$$

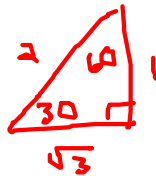
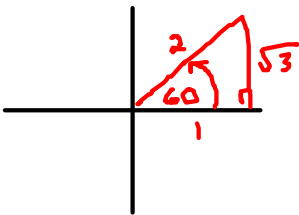
Directions: Find the exact value of each.

6. $\tan[\arcsin(0)]$ $\tan[0^\circ] = \frac{0}{1} = \boxed{0}$



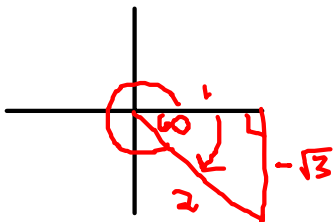
$\sin 0 = 0$
 $\cos 0 = 1$

7. $\sin[\arctan(\sqrt{3})] = \sin 60^\circ$ Tan: Positive I
 Negative IV



$\sin 60 = \boxed{\frac{\sqrt{3}}{2}}$

8. $\arcsin\left[\sin \frac{5\pi}{3}\right]$ $\frac{5\pi}{3} \cdot \frac{180}{\pi} = 300$



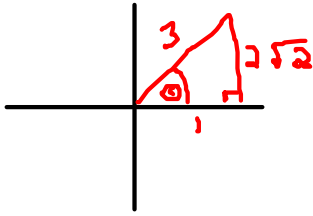
$\sin 300^\circ = -\frac{\sqrt{3}}{2}$

$\arcsin\left(-\frac{\sqrt{3}}{2}\right)$

$[-\pi/2, \pi/2]$

$\cancel{60^\circ} \times \frac{\pi}{\cancel{180}} = \boxed{-\frac{\pi}{3}}$

$$9. \tan \left[\arccos \left(\frac{1}{3} \right) \right]$$

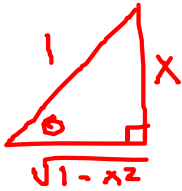


cos: Positive I
Negative II

$$\begin{aligned} a^2 + b^2 &= c^2 \\ 1^2 + b^2 &= 3^2 \\ 1 + b^2 &= 9 \\ -1 & \quad -1 \\ \hline \sqrt{b^2} &= \sqrt{8} \\ b &= \sqrt{8} = \sqrt{4 \cdot 2} = 2\sqrt{2} \end{aligned}$$

$$\tan \theta = \frac{2\sqrt{2}}{1} = \boxed{2\sqrt{2}}$$

$$10. \cos \left[\arcsin x \right]$$



$$\begin{aligned} a^2 + b^2 &= c^2 \\ x^2 + b^2 &= 1^2 \\ x^2 + b^2 &= 1 \\ -x^2 & \quad -x^2 \\ \hline \sqrt{b^2} &= \sqrt{1-x^2} \\ b &= \sqrt{1-x^2} \end{aligned}$$

$$\cos \theta = \frac{\sqrt{1-x^2}}{1} = \boxed{\sqrt{1-x^2}}$$