

Verifying Trigonometric Identities

Reciprocal Identities

$$\begin{aligned}\sin a &= \frac{1}{\csc a} & \csc a &= \frac{1}{\sin a} \\ \cos a &= \frac{1}{\sec a} & \sec a &= \frac{1}{\cos a} \\ \tan a &= \frac{1}{\cot a} & \cot a &= \frac{1}{\tan a}\end{aligned}$$

Quotient Identities

$$\begin{aligned}\tan a &= \frac{\sin a}{\cos a} \\ \cot a &= \frac{\cos a}{\sin a}\end{aligned}$$

Pythagorean Identities

$$\begin{aligned}\sin^2 a + \cos^2 a &= 1 \\ 1 + \tan^2 a &= \sec^2 a \\ 1 + \cot^2 a &= \csc^2 a\end{aligned}$$

Cofunction Identities

$$\begin{aligned}\sin\left(\frac{\pi}{2} - a\right) &= \cos a & \csc\left(\frac{\pi}{2} - a\right) &= \sec a & \tan\left(\frac{\pi}{2} - a\right) &= \cot a \\ \cos\left(\frac{\pi}{2} - a\right) &= \sin a & \sec\left(\frac{\pi}{2} - a\right) &= \csc a & \cot\left(\frac{\pi}{2} - a\right) &= \tan a\end{aligned}$$

Even/Odd Identities

$$\begin{aligned}\sin(-a) &= -\sin a & \csc(-a) &= -\csc a \\ \cos(-a) &= \cos a & \sec(-a) &= \sec a \\ \tan(-a) &= -\tan a & \cot(-a) &= -\cot a\end{aligned}$$

Step 1: Write everything in terms of $\sin x$ and $\cos x$.

Step 2: Use algebra to simplify.

Step 3: If necessary, use $\sin^2 x + \cos^2 x = 1$.

Directions: Use the trigonometric identities to simplify each expression.

1. $\cos x \cdot \tan x = \frac{\cos x}{1} \cdot \frac{\sin x}{\cos x} = \boxed{\sin x}$

$\tan x = \frac{\sin x}{\cos x}$

2. $\sec^2 x (1 - \sin^2 x) = \frac{1}{\cos^2 x} \cdot \frac{(1 - \sin^2 x)}{1} = \frac{1 - \sin^2 x}{\cos^2 x} = \frac{\cos^2 x}{\cos^2 x} = \boxed{1}$

$\sec^2 x = \frac{1}{\cos^2 x}$

$\cancel{\sin^2 x} + \cos^2 x = 1$

$\cancel{-\sin^2 x} - \sin^2 x$

$\cos^2 x = 1 - \sin^2 x$

$$3. \frac{\sec x}{\csc x} = \frac{\frac{1}{\cos x}}{\frac{1}{\sin x}} = \frac{1}{\cos x} \cdot \frac{1}{\sin x} = \frac{1}{\cos x} \cdot \frac{\sin x}{1} = \frac{\sin x}{\cos x} = \boxed{\tan x}$$

$$\sec x = \frac{1}{\cos x} \quad \csc x = \frac{1}{\sin x}$$

$$4. \cot\left(\frac{\pi}{2} - x\right) \cos x = \frac{\sin x}{\cos x} \cdot \frac{\cos x}{1} = \boxed{\sin x}$$

$$\cot\left(\frac{\pi}{2} - x\right) = \tan x$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$5. \underbrace{\sec^2 x \cdot \tan^2 x}_{GCF = \sec^2 x} + \sec^2 x = \sec^2 x (\tan^2 x + 1) = \sec^2 x \cdot \sec^2 x = \boxed{\sec^4 x}$$

$$6. \frac{\csc^2 x - 1}{\csc x - 1} = \frac{(\csc x - 1)(\csc x + 1)}{\csc x - 1} = \boxed{\csc x + 1}$$

$$7. \sin^4 x - \cos^4 x = \frac{(\sin^2 x + \cos^2 x)(\sin^2 x - \cos^2 x)}{\sin^2 x + \cos^2 x = 1} = \boxed{\sin^2 x - \cos^2 x}$$

Directions: Verify each identity.

8. $\cos x \cdot \sec x - \cos^2 x = \sin^2 x$

$$\sec x = \frac{1}{\cos x}$$

$$\frac{\cos x \cdot \frac{1}{\cos x} - \cos^2 x}{1} = 1 - \cos^2 x$$

$$1 - \cos^2 x$$

$$\sin^2 x + \cos^2 x = 1$$
$$-\cos^2 x - \cos^2 x$$

$$\sin^2 x = 1 - \cos^2 x$$

$$\sin^2 x = \sin^2 x \checkmark$$

9. $\frac{\sec^2 x - \tan^2 x + \tan x}{\sec x} = \cos x + \sin x$

$$\tan^2 x + 1 = \sec^2 x$$

$$\frac{\tan^2 x + 1 - \tan^2 x + \tan x}{\sec x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{1 + \tan x}{\sec x}$$

$$\sec x = \frac{1}{\cos x}$$

$$\frac{\cos x \cdot 1 + \frac{\sin x}{\cos x}}{\cos x \cdot 1}$$
$$\frac{1}{\cos x}$$

$$\frac{\frac{\cos x}{\cos x} + \frac{\sin x}{\cos x}}{\frac{1}{\cos x}}$$

$$\cos x + \sin x = \cos x + \sin x \checkmark$$

$$10. \frac{\tan x \cdot \tan x}{\tan x \cdot 1 + \sec x} + \frac{1 + \sec x \cdot (1 + \sec x)}{\tan x \cdot (1 + \sec x)} = 2 \csc x$$

$$LCD = (1 + \sec x)(1 + \tan x)$$

$$\tan^2 x + 1 = \sec^2 x$$

$$\frac{\tan^2 x}{\tan x (1 + \sec x)} + \frac{(1 + \sec x)(1 + \sec x)}{\tan x (1 + \sec x)}$$

$$\frac{\cancel{\sec^2 x}}{\tan^2 x + 1 + \sec x + \sec x + \sec^2 x} = \frac{1}{\tan x (1 + \sec x)}$$

$$GCF = 2 \sec x$$

$$\frac{2 \sec^2 x + 2 \sec x}{\tan x (1 + \sec x)}$$

$$\sec x = \frac{1}{\cos x}$$

$$+ \tan x = \frac{\sin x}{\cos x}$$

$$\frac{2 \sec x (\cancel{\sec x} + 1)}{+ \tan x \cancel{(1 + \sec x)}} = \frac{2}{\frac{\cos x}{\sin x}} = \frac{2}{\sin x}$$

$$2 \csc x = 2 \csc x \quad \checkmark$$

$$11. (\sec x - \tan x) \cdot (\csc x + 1) = \cot x$$

$$\sec x \csc x + \sec x - \tan x \csc x - \tan x$$

$$\frac{1}{\cos x} \cdot \frac{1}{\sin x} + \frac{1}{\cos x} - \frac{\cancel{\sin x} \cdot \frac{1}{\cos x}}{\cancel{\cos x}} - \frac{\sin x}{\cos x}$$

$$\frac{1}{\cos x \sin x} + \frac{1}{\cos x} - \frac{1}{\cos x} - \frac{\sin x}{\cos x}$$

$$\frac{1}{\cos x \sin x} - \frac{\sin x \cdot \frac{\sin x}{\cos x}}{\cos x \cdot \sin x} \quad LCD = \cos x \cdot \sin x$$

$$\frac{1}{\cos x \sin x} - \frac{\sin^2 x}{\cos x \sin x} = \frac{1 - \sin^2 x}{\cos x \sin x} =$$

$$\sec x = \frac{1}{\cos x}$$

$$\csc x = \frac{1}{\sin x}$$

$$\tan x = \frac{\sin x}{\cos x}$$

$$\frac{\cos^2 x}{\cos x \sin x} = \frac{\cos x}{\sin x}$$

$$\cot x = \cot x \quad \checkmark$$

$$\cot x = \cot x \checkmark$$

12. $\cos^3 x \cdot \sin^2 x = (\sin^2 x - \sin^4 x) \cdot \cos x$

$\sin^2 x + \cos^2 x = 1$

$\sin^2 x = 1 - \cos^2 x$

Left: $\frac{\cos^3 x (1 - \cos^2 x)}{\cos^3 x - \cos^5 x}$

Right: $\left[1 - \cos^2 x - (1 - \cos^2 x)^2 \right] \cos x$

$\left[1 - \cos^2 x - (1 - \cos^2 x)(1 - \cos^2 x) \right] \cos x$

$1 - \cos^2 x - \cos^2 x + \cos^4 x$

$1 - 2\cos^2 x + \cos^4 x$

$[x - \cos^2 x - x + 2\cos^2 x - \cos^4 x] \cos x$

$[\cos^2 x - \cos^4 x] \cos x$

$\frac{\cos^3 x - \cos^5 x}{\cos^3 x - \cos^5 x} \checkmark$

13. $\frac{1 - \sin x}{\sin x \cdot \cot x} = \frac{\cos x}{1 + \sin x}$

$\cot x = \frac{\cos x}{\sin x}$

$$\begin{aligned} \frac{1 - \sin x}{\sin x \cdot \frac{\cos x}{\sin x}} &= \frac{1 - \sin x}{\cos x} \cdot \frac{1 + \sin x}{1 + \sin x} = \frac{(1 - \sin x)(1 + \sin x)}{\cos x(1 + \sin x)} \\ &= \frac{1 + \sin x - \sin x - \sin^2 x}{\cos x(1 + \sin x)} = \frac{\cos^2 x}{\cos x(1 + \sin x)} = \frac{\cos x}{1 + \sin x} \\ &\underline{\underline{\cos x}} = \underline{\underline{\cos x}} \checkmark \end{aligned}$$

$$\frac{\cos x}{1 + \sin x} = \frac{\cos x}{1 + \sin x} \quad \checkmark$$