Solving Trigonometric Equations

Solve each equation for $0 \le x < 2\pi$.

1. $\sqrt{2}\cos x + 1 = 0$

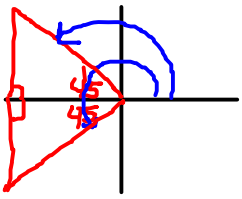
$-1 \quad -1$

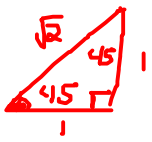
$\frac{\sqrt{2}\cos x}{\sqrt{2}} = \frac{-1}{\sqrt{2}}$

$\cos x = \frac{-1}{\sqrt{2}}$

Reference $\measuredangle = 45°$

II III





II: $180 - 45 = 135$

$135 \cdot \frac{\pi}{180} = \frac{3\pi}{4}$

III: $180 + 45 = 225°$

$225 \cdot \frac{\pi}{180} = \frac{5\pi}{4}$

$\left\{3\pi/4, 5\pi/4\right\}$

2. $3\sec^2 x - 4 = 0$

$+4 \quad +4$

$\frac{3\sec^2 x}{3} = \frac{4}{3}$

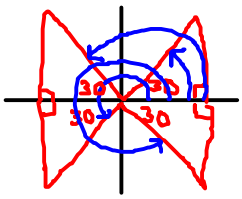
$\sqrt{\sec^2 x} = \sqrt{\frac{4}{3}}$

$\sec x = \pm \frac{2}{\sqrt{3}}$

$\cos x = \pm \frac{\sqrt{3}}{2}$

Reference $\measuredangle$: 30°

I IV II III



I: $30 \cdot \frac{\pi}{180} = \frac{\pi}{6}$

II: $180 - 30 = 150 \cdot \frac{\pi}{180} = \frac{5\pi}{6}$

III: $180 + 30 = 210 \cdot \frac{\pi}{180} = \frac{7\pi}{6}$

IV: $360 - 30 = 330 \cdot \frac{\pi}{180} = \frac{11\pi}{6}$

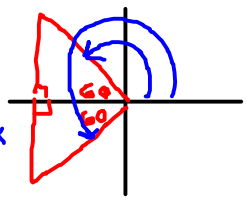
$\left\{\pi/6, 5\pi/6, 7\pi/6, 11\pi/6\right\}$

3. $2\sin^2 x = 2 + \cos x$

$\sin^2 x + \cos^2 x = 1$

$-\cos^2 x \quad -\cos^2 x$

$\sin^2 x = 1 - \cos^2 x$



$2(1-\cos^2 x) = 2 + \cos x$

$2 - 2\cos^2 x = 2 + \cos x$

$+2\cos^2 x \quad +2\cos^2 x$

$2\cos^2 x + \cos x = 0$ GCF = cos x

$\cos x(2\cos x + 1) = 0$

$\cos x = 0$

$2\cos x + 1 = 0$

$-1 \quad -1$

$\frac{2\cos x}{2} = \frac{-1}{2}$

$\cos x = -1/2$

Reference ∠ = 60°

II III

$90° \cdot \frac{\pi}{180} = \frac{\pi}{2}$

$270° \cdot \frac{\pi}{180} = \frac{3\pi}{2}$

II: $180 - 60 = 120 \cdot \frac{\pi}{180} = \frac{2\pi}{3}$

III: $180 + 60 = 240 \cdot \frac{\pi}{180} = \frac{4\pi}{3}$

$\{\pi/2, 3\pi/2, 2\pi/3, 4\pi/3\}$

4. $2\cos(3x) - 1 = 0$

$+1 \quad +1$

$\frac{2\cos 3x}{2} = \frac{1}{2}$

$\cos 3x = \frac{1}{2}$

Ref. ∠ = 60°

I: 60

IV: 300

$3x = 60$ $3x = 300$

$x = 20$ $x = 100$

$20° \times \frac{\pi}{180} = \frac{\pi}{9}$

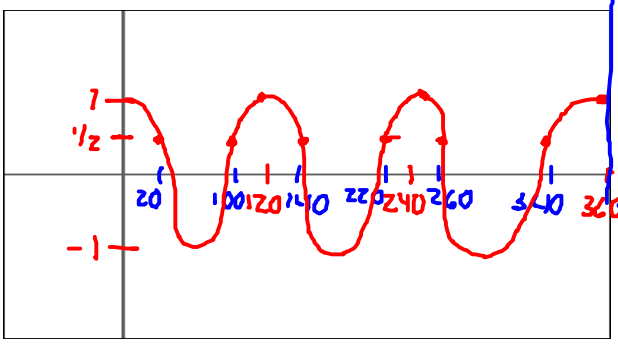
$100° \cdot \frac{\pi}{180} = \frac{5\pi}{9}$

$140° \cdot \frac{\pi}{180} = \frac{7\pi}{9}$

$220° \cdot \frac{\pi}{180} = \frac{11\pi}{9}$

$260° \cdot \frac{\pi}{180} = \frac{13\pi}{9}$

$340° \cdot \frac{\pi}{180} = \frac{17\pi}{9}$



$\{\pi/9, 5\pi/9, 7\pi/9, 11\pi/9, 13\pi/9, 17\pi/9\}$

5. $\cos x + \sin x \tan x = 2$

$\tan x = \frac{\sin x}{\cos x}$

$\cos x + \sin x \cdot \frac{\sin x}{\cos x} = 2$

$\cos x \cdot \frac{\cos x}{\cos x \cdot 1} + \frac{\sin^2 x}{\cos x} = 2$

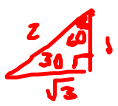
$\frac{\cos^2 x + \sin^2 x}{\cos x} = 2$ ($\cos^2 x + \sin^2 x = 1$)

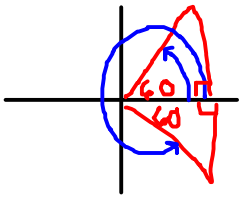
$\frac{1}{\cos x} = 2$

$\sec x = 2$

$\cos x = \frac{1}{2}$

Ref. $\angle$: $60°$





I: $60° \cdot \frac{\pi}{180} = \frac{\pi}{3}$

IV: $300° \cdot \frac{\pi}{180} = \frac{5\pi}{3}$

$\{\pi/3, 5\pi/3\}$

6. $\csc^2 x - 4\cot x = -2$

$1 + \cot^2 x = \csc^2 x$

$1 + \cot^2 x - 4\cot x = -2$

$+2 \qquad +2$

$\cot^2 x - 4\cot x + 3 = 0$

$(\cot x - 1)(\cot x - 3) = 0$

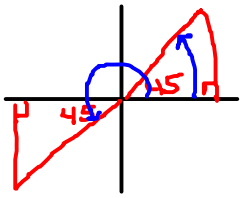
$\cot x - 1 = 0 \qquad \cot x - 3 = 0$

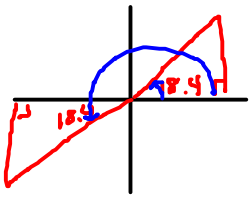
$\cot x = 1 \qquad \cot x = 3$

$\tan x = 1 \qquad \tan x = \frac{1}{3}$

Ref $\angle$: $18.4°$

Ref. $\angle$: $45°$





I: $45°$

III: $180 + 45 = 225°$

I: $18.4°$

III: $180 + 18.4 = 198.4°$

$\{45°, 225°, 18.4°, 198.4°\}$