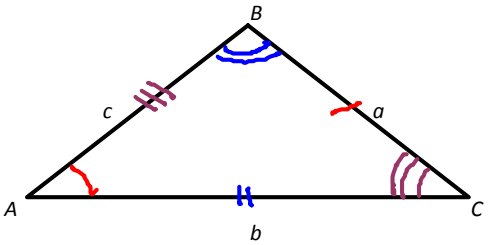
Law of Sines

The Law of Sines is used to solve oblique triangles (triangles that do not have a right angle) when you have an angle-side pair.



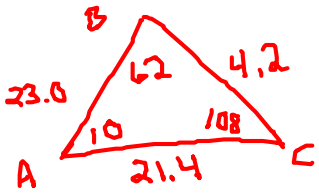
$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

1. Solve the triangle.

$\measuredangle A = 10^\circ$

$a = 4.2$

$\measuredangle B = 62^\circ$



$\measuredangle C$

$$\begin{array}{r} 10 \\ +62 \\ \hline 72 \end{array} \qquad \begin{array}{r} 180 \\ -72 \\ \hline 108 \end{array}$$

$\boxed{\measuredangle C = 108^\circ}$

side b

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 10}{4.2} = \frac{\sin 62}{b}$$

$$b \cdot \frac{\cancel{\sin 10}}{\cancel{\sin 10}} = \frac{4.2 \sin 62}{\sin 10}$$

$\boxed{b = 21.4}$

side c

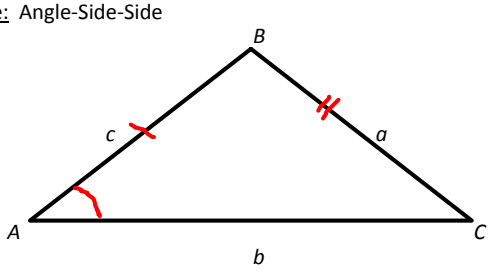
$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 10}{4.2} = \frac{\sin 108}{c}$$

$$c \cdot \frac{\cancel{\sin 10}}{\cancel{\sin 10}} = \frac{4.2 \sin 108}{\sin 10}$$

$\boxed{c = 23.0}$

The Ambiguous Case: Angle-Side-Side

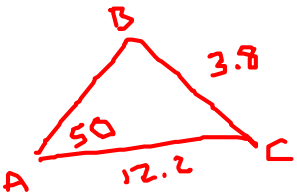


No Triangle
One Triangle
Two Triangles

2. Solve each triangle, if possible. If two triangles exist, find both triangles.

a) No Triangle

$\measuredangle A = 50^\circ$
$a = 3.8$
$b = 12.2$



A-S-S

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 50}{3.8} = \frac{\sin B}{12.2}$$

$$\frac{3.8 \sin B}{3.8} = \frac{12.2 \sin 50}{3.8}$$

$$\sin B = 2.4594$$

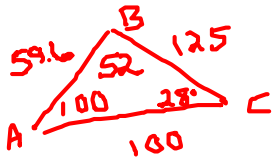
$\boxed{\text{The } \triangle \text{ does not exist}}$

b) <u>One Triangle</u>

$\measuredangle A = 100^\circ$

$a = 125$

$b = 100$



A-S-S

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 100}{125} = \frac{\sin B}{100}$$

$$\frac{125 \sin B}{125} = \frac{100 \sin 100}{125}$$

$\sin B = .7878$

$\measuredangle B = 52^\circ$

$\measuredangle C = 28^\circ$

<u>Check for 2nd △</u>

$\measuredangle A = 100^\circ$

$\measuredangle B = 180 - 52 = 128^\circ$

2nd △ does not exist because $100 + 128 > 180$

<u>side C</u>

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 100}{125} = \frac{\sin 28}{c}$$

$$\frac{c \cdot \sin 100}{\sin 100} = \frac{125 \cdot \sin 28}{\sin 100}$$

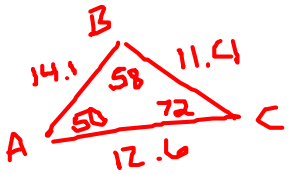
$\boxed{c = 59.6}$

c) Two Triangles

$\measuredangle A = 50^\circ$

$a = 11.4$

$b = 12.6$



A - S - S

1st △

$$\frac{\sin A}{a} = \frac{\sin B}{b}$$

$$\frac{\sin 50}{11.4} = \frac{\sin B}{12.6}$$

$$\frac{11.4 \sin B}{11.4} = \frac{12.6 \sin 50}{11.4}$$

$\sin B = .8467$

$\measuredangle B = 58^\circ$ $\measuredangle C = 72^\circ$

Side c

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 50}{11.4} = \frac{\sin 72}{c}$$

$$c \cdot \frac{\sin 50}{\sin 50} = \frac{11.4 \sin 72}{\sin 50}$$

$\boxed{c = 14.1}$

Check 2nd △

$\measuredangle A = 50$

$\measuredangle B = 180 - 58 = 122$

$\measuredangle C = 8$

Side c

$$\frac{\sin A}{a} = \frac{\sin C}{c}$$

$$\frac{\sin 50}{11.4} = \frac{\sin 8}{c}$$

$$c \cdot \frac{\sin 50}{\sin 50} = \frac{11.4 \sin 8}{\sin 50}$$

$\boxed{c = 2.1}$