Proofs of Derivatives of Trigonometric Functions

To prove the derivatives of trigonometric functions, use the definition of the derivative and the trigonometric identities.

Definition of the Derivative:

$$\lim_{h\to 0}\frac{f(x+h)-f(x)}{h}$$

Special Limits:

$$\lim_{x\to 0}\frac{\sin x}{x}=1$$

$$\lim_{x\to 0}\frac{1-\cos x}{x}=0$$

$$\lim_{x\to 0}\frac{\cos x-1}{x}=0$$

Trigonometric Identities:

$$\sin^2 x+\cos^2 x=1$$

$$\sin(x+y)=\sin x\cos y+\cos x\sin y$$

$$\cos(x+y)=\cos x\cos y-\sin x\sin y$$

$$\tan x=\frac{\sin x}{\cos x}$$

$$\cot x=\frac{\cos x}{\sin y}$$

$$\sec x=\frac{1}{\cos x}$$

$$\csc x=\frac{1}{\sin x}$$

Proof: $\frac{d}{dx}\sin x=\cos x$

Proof: $\frac{d}{dx}\cos x=-\sin x$

Proof: $\frac{d}{dx}\tan x = \sec^2 x$

Proof: $\frac{d}{dx}\sec x = \sec x \tan x$

Proof: $\frac{d}{dx}\csc x = -\csc x \cot x$

Proof: $\dfrac{d}{dx}\cot x = -\csc^2 x$