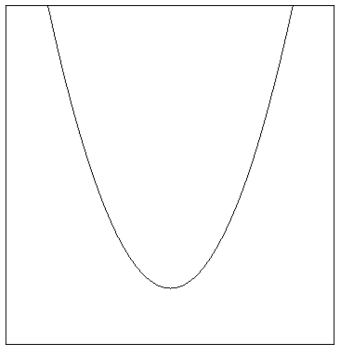
Concavity and the Second Derivative Test

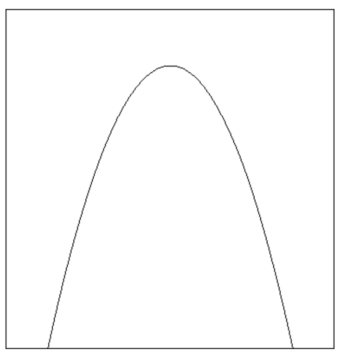
Concavity

A function is concave up on an interval if the graph lies above the horizontal tangent.



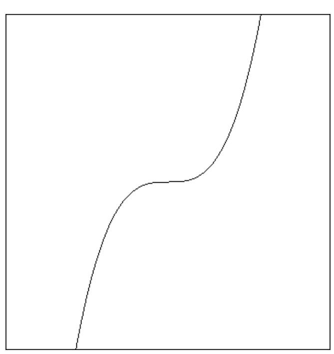
$f''(x_0) > 0$

A function is concave down on an interval if the graph lies below the horizontal tangent.



$f''(x_0) < 0$

Inflection Point - A point on the graph where the concavity changes.



$f''(x_0) = 0$ or $f''(x_0) = DNE$

Second Derivative Test

Let x_0 be a critical point.

If $f'(x_0)=0$ and $f''(x_0)<0$ then $(x_0, f(x_0))$ is a relative maximum.

If $f'(x_0)=0$ and $f''(x_0)>0$ then $(x_0, f(x_0))$ is a relative minimum.

If $f'(x_0)=0$ and $f''(x_0)=0$ then the test fails.

1. Find the points of inflection and determine the concavity of the graph of the function.

   a) $f(x)=2x^4-8x+3$

b) $f(x) = x\sqrt{x+1}$

c) $f(x) = \frac{1}{3}x^3 - \frac{1}{2}x^2 - 2x + \frac{1}{3}$

2. Find the relative extrema using the Second Derivative Test.

a) $f(x) = 3x^4 - 4x^3 - 12x^2$

b) $f(x) = \sqrt{x^2 + 1}$

3. A function f is continuous on $[-3,3]$ such that $f(-3)=4$ and $f(3)=1$. The functions f' and f'' have the properties given in the table below. Sketch the graph.

	f'	f''
$-3<x<-1$	+	+
$x=-1$	*DNE*	*DNE*
$-1<x<1$	−	+
$x=1$	0	0
$1<x<3$	−	−