Areas of Surfaces of Revolution

The area of a surface $f(x)$ rotated about the x-axis:

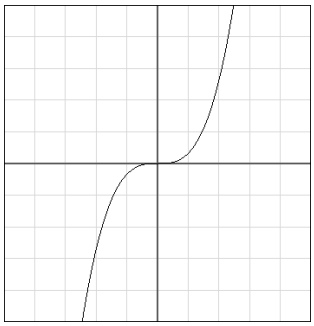
$$S = \int_a^b 2\pi \bullet \text{radius} \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

The area of a surface $g(y)$ rotated about the y-axis:

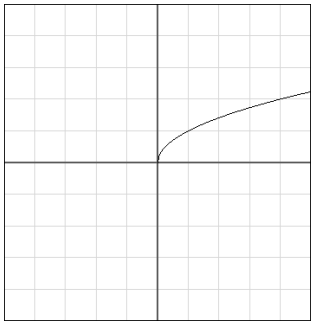
$$S = \int_c^d 2\pi \bullet \text{radius} \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy$$

Directions: For questions 1 through 3, find the area of the surface obtained by rotating the curve about the x-axis.

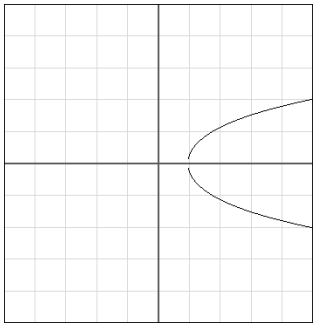
1. $y = \frac{1}{3}x^3,\ 0 \le x \le 2$



2. $y = \sqrt{x},\ \left[\frac{3}{4}, 2\right]$

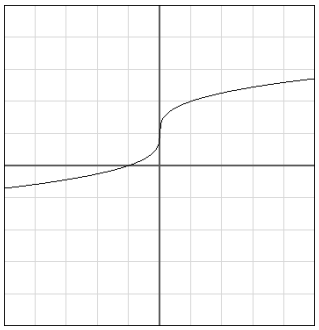


3. $x = \frac{1}{3}\left(y^2 + 2\right)^{\frac{3}{2}},\ 1 \le y \le 2$



Directions: For questions 4 and 5, find the area of the surface obtained by rotating the curve about the *y*-axis.

4. $y = \sqrt[3]{x} + 1, \ [0, 4]$



5. $x = \dfrac{y^2 - \ln y}{2\sqrt{2}}$, $1 \le y \le 3$