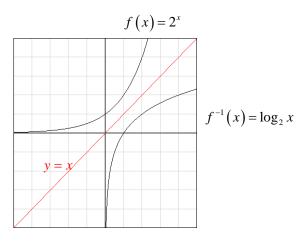
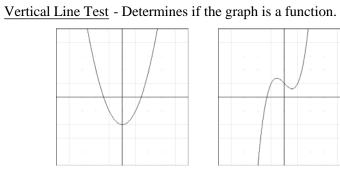
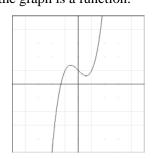
## **Inverse Functions**

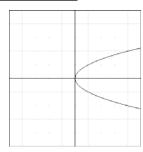


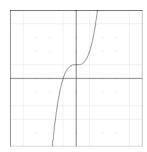
$$f(x) = 2^x$$



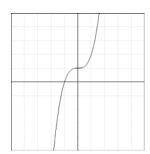


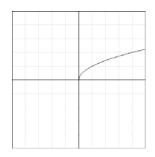
<u>Horizontal Line Test</u> - Determines if the function is one-to-one.

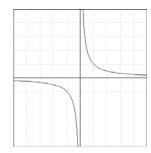




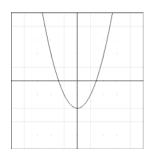
One-to-One Function - A graph that passes both the vertical and horizontal line tests. The graph is a function and it has an inverse.



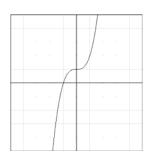




Monotonic Function - A function that is always increasing or always decreasing.



$$f(x) = x^2 - 2$$



$$f\left(x\right) = x^3 + 1$$

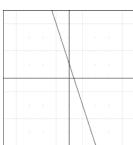
If f(x) and g(x) are inverse functions, then f(g(x)) = g(f(x)) = x.

$$f\left(x\right) = x^3$$

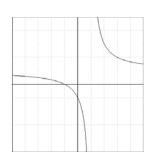
$$g(x) = \sqrt[3]{x}$$

Directions: For questions 1 through 7, determine if the function has an inverse.

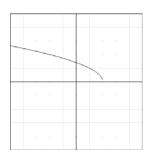
1. 
$$f(x) = -3x + 1$$



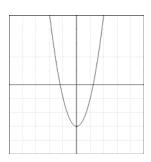
$$2. f(x) = \frac{x+1}{x-1}$$



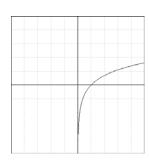
$$3. f(x) = \sqrt{2-x}$$



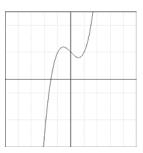
4. 
$$f(x) = 2x^2 - 3$$



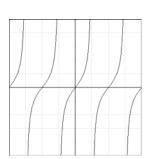
$$5. f(x) = \ln x$$



6. 
$$f(x) = x^3 - x + 2$$



$$7. f(x) = \tan x$$



Directions: For questions 8 through 12, find the inverse of each function.

8. 
$$f(x) = -3x + 1$$

9. 
$$f(x) = \frac{3x+1}{2x-1}$$

10. 
$$f(x) = \sqrt{2-x}$$

11. 
$$f(x) = 2x^2 - 3, x \ge 0$$

 $12. \ f(x) = \ln x$ 

Directions: For question 13, show that the functions are inverses of each other.

13. 
$$f(x) = \frac{1}{x-2} + 1$$

$$g\left(x\right) = \frac{2x-1}{x-1}$$

Derivative of the Inverse Function:  $(f^{-1})'(x) = \frac{1}{f'(g(x))}$ 

Steps to find the derivative of the inverse at x = a.

- 1. Set f(x) equal to a and solve for x.
- 2. Find f'(x) at the value of x found in step 1.
- 3. Take the recriprocal of the value found in step 2.

Directions: For questions 14 and 15, find  $(f^{-1})'(a)$ .

14. 
$$f(x) = \frac{\ln e^{3x}}{x-1}$$
,  $a = 2$ 

15. 
$$f(x) = x^3 - x^2 + 2x + 2$$
,  $a = 4$