Hyperbolic Functions

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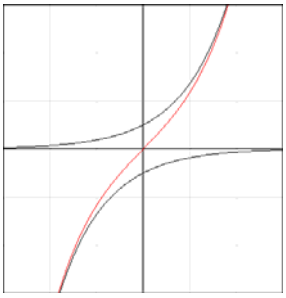
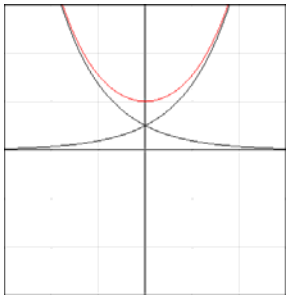


$\sinh x = \dfrac{e^x - e^{-x}}{2}$

Domain: $(-\infty, \infty)$

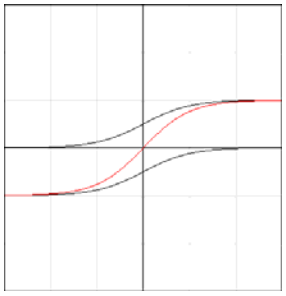
Range: $(-\infty, \infty)$



$\cosh x = \dfrac{e^x + e^{-x}}{2}$

Domain: $(-\infty, \infty)$

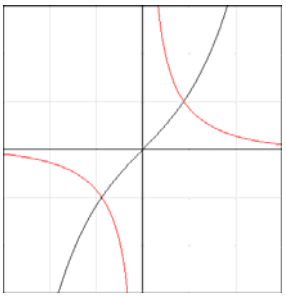
Range: $[1, \infty)$



$\tanh x = \dfrac{\sinh x}{\cosh x} = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$

Domain: $(-\infty, \infty)$

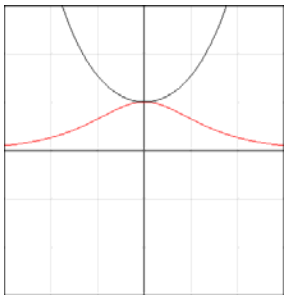
Range: $(-1,1)$



$\operatorname{csch} x = \dfrac{1}{\sinh x} = \dfrac{2}{e^x - e^{-x}}$

Domain: $(-\infty, 0) \cup (0, \infty)$

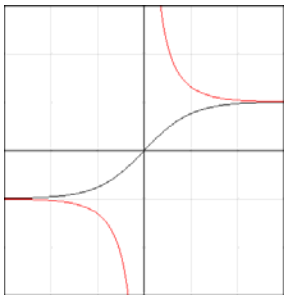
Range: $(-\infty, 0) \cup (0, \infty)$



$\operatorname{sech} x = \dfrac{1}{\cosh x} = \dfrac{2}{e^x + e^{-x}}$

Domain: $(-\infty, \infty)$

Range: $(0,1]$



$\coth x = \dfrac{\cosh x}{\sinh x} = \dfrac{e^x + e^{-x}}{e^x - e^{-x}}$

Domain: $(-\infty, 0) \cup (0, \infty)$

Range: $(-\infty, -1) \cup (1, \infty)$

Hyperbolic Identities

$\cosh^2 x - \sinh^2 x = 1$

$\tanh^2 x + \operatorname{sech}^2 x = 1$

$\coth^2 x - \operatorname{csch}^2 x = 1$

$\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

$\cosh^2 x = \dfrac{\cosh 2x + 1}{2}$

$\sinh 2x = 2\sinh x \cosh x$

$\cosh 2x = \cosh^2 x + \sinh^2 x$

$\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

$\sinh(x - y) = \sinh x \cosh y - \cosh x \sinh y$

$\cosh(x + y) = \cosh x \cosh y + \sinh x \sinh y$

$\cosh(x - y) = \cosh x \cosh y + \sinh x \sinh y$

Derivatives of Hyperbolic Functions

$\dfrac{d}{dx}(\sinh x) = \cosh x$

$\dfrac{d}{dx}(\cosh x) = \sinh x$

$\dfrac{d}{dx}(\tanh x) = \operatorname{sech}^2 x$

$\dfrac{d}{dx}(\operatorname{csch} x) = -\operatorname{csch} x \coth x$

$\dfrac{d}{dx}(\operatorname{sech} x) = -\operatorname{sech} x \tanh x$

$\dfrac{d}{dx}(\coth x) = -\operatorname{csch}^2 x$

Integrals of Hyperbolic Functions

$\int \cosh x \, dx = \sinh x + C$

$\int \sinh x \, dx = \cosh x + C$

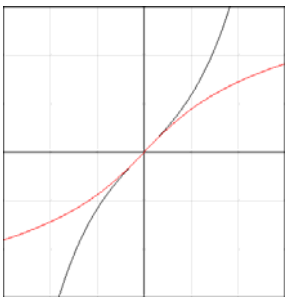
$\int \operatorname{sech}^2 x \, dx = \tanh x + C$

$\int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x + C$

$\int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x + C$

$\int \operatorname{csch}^2 x \, dx = -\coth x + C$

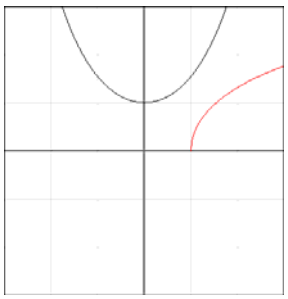
Inverse Hyperbolic Functions



$\sinh^{-1} x = \ln\left(x + \sqrt{x^2 + 1}\right)$

Domain: $(-\infty, \infty)$

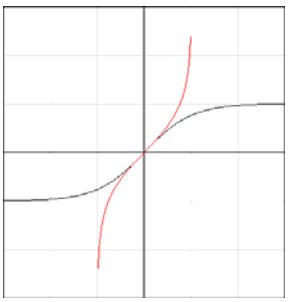
Range: $(-\infty, \infty)$



$\cosh^{-1} x = \ln\left(x + \sqrt{x^2 - 1}\right)$

Domain: $[1, \infty)$

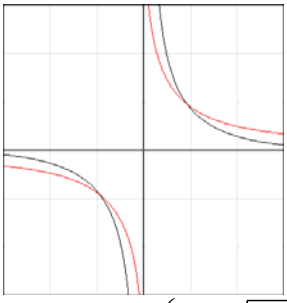
Range: $[0, \infty)$



$\tanh^{-1} x = \frac{1}{2}\ln\left(\frac{1+x}{1-x}\right)$

Domain: $(-1, 1)$

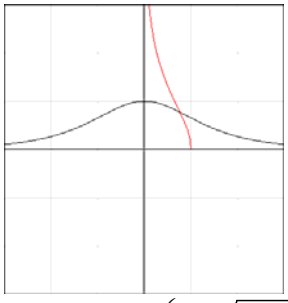
Range: $(-\infty, \infty)$



$\operatorname{csch}^{-1} x = \ln\left(\frac{1}{x} + \frac{\sqrt{1+x^2}}{|x|}\right)$

Domain: $(-\infty, 0) \cup (0, \infty)$

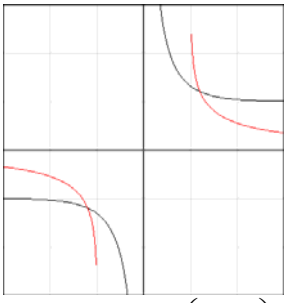
Range: $(-\infty, 0) \cup (0, \infty)$



$\operatorname{sech}^{-1} x = \ln\left(\frac{1+\sqrt{1-x^2}}{x}\right)$

Domain: $(0, 1]$

Range: $[0, \infty)$



$\coth^{-1} x = \frac{1}{2}\ln\left(\frac{x+1}{x-1}\right)$

Domain: $(-\infty, -1) \cup (1, \infty)$

Range: $(-\infty, 0) \cup (0, \infty)$

Derivatives of Inverse Hyperbolic Functions

$$\frac{d}{dx}\left(\sinh^{-1} x\right) = \frac{1}{\sqrt{x^2+1}}$$

$$\frac{d}{dx}\left(\cosh^{-1} x\right) = \frac{1}{\sqrt{x^2-1}}$$

$$\frac{d}{dx}\left(\tanh^{-1} x\right) = \frac{1}{1-x^2}$$

$$\frac{d}{dx}\left(\operatorname{csch}^{-1} x\right) = -\frac{1}{|x|\sqrt{1+x^2}}$$

$$\frac{d}{dx}\left(\operatorname{sech}^{-1} x\right) = -\frac{1}{x\sqrt{1-x^2}}$$

$$\frac{d}{dx}\left(\coth^{-1} x\right) = \frac{1}{1-x^2}$$

Integrals of Inverse Hyperbolic Functions

$$\int \frac{1}{\sqrt{1+x^2}}\,dx = \sinh^{-1} x + C$$

$$\int \frac{1}{\sqrt{x^2-1}}\,dx = \cosh^{-1} x + C$$

$$\int \frac{1}{1-x^2}\,dx = \tanh^{-1} x + C$$

$$\int \frac{1}{x\sqrt{1+x^2}}\,dx = -\operatorname{csch}^{-1}|x| + C$$

$$\int \frac{1}{x\sqrt{1-x^2}}\,dx = -\operatorname{sech}^{-1}|x| + C$$

$$\int \frac{1}{1-x^2}\,dx = \coth^{-1} x + C$$

Directions: For questions 1 through 6, find the value of each expression.

1. $\sinh 0$

2. $\cosh 0$

3. $\tanh 0$

4. $\sinh^{-1} 0$

5. $\cosh^{-1} 0$

6. $\tanh^{-1} 0$

Directions: For questions 7 and 8, verify each identity.

7. $\sinh^2 x = \dfrac{\cosh 2x - 1}{2}$

8. $\sinh(x + y) = \sinh x \cosh y + \cosh x \sinh y$

Directions: For questions 9 through 11, find each limit.

9. $\lim_{x \to \infty} \sinh x$

10. $\lim_{x \to 0^-} \coth x$

11. $\lim_{x \to 0^+} \coth x$

Directions: For questions 12 and 13, find the values of the five remaining trigonometric functions.

12. $\sinh x = \frac{4}{3}$

13. $\coth x = 2$

Directions: For questions 14 through 18, find each derivative.

14. $y = \sinh 5x$

15. $y = \ln\left(\tanh 2x\right)$

16. $y = \tan^{-1}\left(\sinh x\right)$

17. $y = \tanh^{-1}\left(\sinh x\right)$

18. $y = \left(\operatorname{sech}^{-1} 2x\right)^2$

Directions: For questions 19 through 23, evaluate each integral.

19. $\int_0^{\frac{1}{2}} e^x \cosh x \, dx$

20. $\int_0^{\ln 3} \tanh x \, dx$

21. $\int \cosh^2 x \, dx$

22. $\int \frac{1}{16-x^2}\,dx$

23. $\int \frac{1}{\sqrt{1-e^{2x}}}\,dx$