Verifying Trigonometric Identities

Reciprocal Identities

$\sin a = \dfrac{1}{\csc a}$ $\quad \csc a = \dfrac{1}{\sin a}$

$\cos a = \dfrac{1}{\sec a}$ $\quad \sec a = \dfrac{1}{\cos a}$

$\tan a = \dfrac{1}{\cot a}$ $\quad \cot a = \dfrac{1}{\tan a}$

Quotient Identities

$\tan a = \dfrac{\sin a}{\cos a}$

$\cot a = \dfrac{\cos a}{\sin a}$

Pythagorean Identities

$\sin^2 a + \cos^2 a = 1$

$1 + \tan^2 a = \sec^2 a$

$1 + \cot^2 a = \csc^2 a$

Cofunction Identities

$\sin\left(\dfrac{\pi}{2} - a\right) = \cos a$ $\quad \csc\left(\dfrac{\pi}{2} - a\right) = \sec a$ $\quad \tan\left(\dfrac{\pi}{2} - a\right) = \cot a$

$\cos\left(\dfrac{\pi}{2} - a\right) = \sin a$ $\quad \sec\left(\dfrac{\pi}{2} - a\right) = \csc a$ $\quad \cot\left(\dfrac{\pi}{2} - a\right) = \tan a$

Even/Odd Identities

$\sin(-a) = -\sin a$ $\quad \csc(-a) = -\csc a$

$\cos(-a) = \cos a$ $\quad \sec(-a) = \sec a$

$\tan(-a) = -\tan a$ $\quad \cot(-a) = -\cot a$

Step 1: Write everything in terms of $\sin x$ and $\cos x$.

Step 2: Use algebra to simplify.

Step 3: If necessary, use $\sin^2 x + \cos^2 x = 1$.

Directions: Use the trigonometric identities to simplify each expression.

1. $\cos x \bullet \tan x$

2. $\sec^2 x\left(1 - \sin^2 x\right)$

3. $\dfrac{\sec x}{\csc x}$

4. $\cot\left(\dfrac{\pi}{2}-x\right)\cos x$

5. $\sec^2 x \bullet \tan^2 x + \sec^2 x$

6. $\dfrac{\csc^2 x - 1}{\csc x - 1}$

7. $\sin^4 x - \cos^4 x$

Directions: Verify each identity.

8. $\cos x \bullet \sec x - \cos^2 x = \sin^2 x$

9. $\dfrac{\sec^2 x - \tan^2 x + \tan x}{\sec x} = \cos x + \sin x$

10. $\dfrac{\tan x}{1+\sec x}+\dfrac{1+\sec x}{\tan x}=2\csc x$

11. $(\sec x-\tan x)\bullet(\csc x+1)=\cot x$

12. $\cos^3 x \bullet \sin^2 x = \left(\sin^2 x - \sin^4 x\right) \bullet \cos x$

13. $\dfrac{1-\sin x}{\sin x \bullet \cot x} = \dfrac{\cos x}{1+\sin x}$